Unit 3 Lab1

What Lies Between the Stars

Teacher Version
Objective: In this lab, students will measure and compare the peak intensity of 21 cm radio waves emitted from the gas and dust between five different objects.

Teacher Notes

PARIPod 3.1 is still in development.

Background Knowledge

On earth and in space we are immersed in a sea of electromagnetic waves. Electromagnetic waves include gamma rays (very high energy light), X-rays (high energy light), ultraviolet rays (light that gives us tans), visible light (light we can see with our eyes), infrared light (light that we sense as heat), microwaves (light we use in our microwaves), and finally radio waves (light we listen to on the radio and on our cell phones). Almost everything emits, reflects and absorbs electromagnetic waves. For more information on electromagnetic waves see Lab 2. In this lab we will focus on nebula or clouds of dust and gas found between stars.

If you go outside with a pair of binoculars and look into the constellation of Orion, you will see more than stars. If you look close enough, you will see a cloud of gas and dust. We call this the Orion Nebula (Figure 1). The Orion Nebula is made mostly of hydrogen gas. In fact 70% of our observable universe is made of hydrogen gas. Consequently, the dominant source of radio waves, which we will be observing with Smiley, comes from hydrogen gas.

So how does hydrogen gas emit radio waves? Here’s the science: A neutral hydrogen atom consists of a proton surrounded by an electron. One property that both an electron and a proton have is something called spin. (Figure 2) Through a crude analogy, spin can be thought of as a spinning earth orbiting a spinning Sun. If we picture spin as this rotation we can place a direction on spin. If the spin of the earth is up, then the earth is spinning counter clockwise as it orbits the sun. If the spin of the earth is down, then it is
spinning clockwise as it orbits the sun. The same works for the sun. When the spins of an electron and proton are in the same direction, meaning they are both spinning in the same direction, then the spins are said to be **parallel**. When the spins of an electron and a proton are in different directions, meaning they are spinning in different directions, the spins are said to be **antiparallel**. When an electron and a proton have parallel spins, the electron is in a slightly higher energy level than when the electron and proton have antiparallel spins. Nature always prefers to be in the lowest energy possible, so parallel spins usually decay into antiparallel spins. Hydrogen atoms in a nebula can collide or absorb light, causing the electron to flip from its natural state of being antiparallel to the proton to parallel to the spin of the proton. Since the electron wants to be in its lowest energy level it flips back to antiparallel. However, when the electron *spin-flips* it emits a radio wave with wavelength of 21 centimeters and a frequency of 1.42 Gigahertz (1 billion Hertz).

Wavelength is simply the length of a cycle and frequency is the number of cycles per second. (See 3.1 for further discussion on wavelength and frequency.) Using Smiley, you will measure 21 cm radio waves from gas and dust that exists between the stars, emitted from the spin-flip transitions from hydrogen. These 21 cm radio waves measure slightly less than the width of an 8 1/2 x 11 sheet of paper. However, these radio waves are much longer than a visible electromagnetic wave (typically visible light has a wavelength of about 550 nanometers which is about 400,000 times smaller than our radio wave).
Procedures:

1. Log into the Smiley Observation Room. For information refer to Unit 2: Smiley Basics.

2. In Map mode select an object by either clicking on its red star or by selecting one by pulling down the Sources tab.

3. Once you have selected your object, click on the GO button. You can follow Smiley’s progress by watching the live video in the top center of your screen. Check the status box and notice when Smiley stops.
   NOTE: Not all objects will be above the horizon. If the Altitude is negative, then the object is below the horizon and cannot be observed. You will get an error message in the status box. (Figure 3)

4. Once Smiley stops, check the coordinates. The Target Coordinates (in the white boxes) should be close to the CURRENT Coordinates. To get closer, you can use the Hand Paddle to point Smiley closer to the desired coordinates.

5. Next click on the Spectrum button, to set Smiley in spectrum mode.

6. In the Spectrum window, set your IF GAIN to 19 and set your PLOT RATE to 1x. Also make sure the Base Frequency is set to 1.42 GHz. Click Begin Scan.

Figure 2. The production of 21cm wavelength radio waves from interstellar hydrogen.
7. After Smiley has completed the plot and starts to plot again, click on **Stop Scan**.
   Record the peak intensity on the table below by clicking on **Save Scan** and saving it to some **Output filename**. Next open your saved scan by clicking on **Open Data File**. Then click on **List**, this will list all of your data points.

8. Now find your peak intensity and record your results in the table below.

9. **Repeat steps 2-7** for four more objects making sure you use the same IF gain and plot rate for each object.

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**Data Collection**

<table>
<thead>
<tr>
<th>Object</th>
<th>Peak Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

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**Analysis and Discussion**

1. How did your spectrum scans compare from object to object?

   *There should be differences in peak intensities for each object.*

2. What could account for the difference?

   *The higher the peak intensity, the more spinflip transitions of hydrogen and thus more radio waves.*
Unit 3 Lab 2

Radio Waves from Space

Teacher Version
Unit 3 Lab 2:
Radio Waves from Space

Objective: In this lab, students will measure and compare the peak intensity of 21 cm radio waves emitted from the gas and dust between five different objects.

Teacher Notes

PARIPod 3.2 is still in development.

Background Knowledge

When we discuss electromagnetic waves we usually think of visible light, but visible light actually makes up only a small fraction of the electromagnetic spectrum. The electromagnetic spectrum is the total range of frequency or wavelengths of electromagnetic waves. The spectrum’s range extends from the long wavelengths of radio waves to the short wavelengths of gamma waves. Below is a diagram of the electromagnetic spectrum (Figure 1).

Electromagnetic waves are similar in many ways to mechanical waves, therefore many of the terms and mathematical equations used for mechanical waves can be used in the study of electromagnetic waves.

Figure 1. The Electromagnetic Spectrum.

For simplicity we can think of an electromagnetic wave as energy that is moving from place to place and travels in the form of a transverse wave. Illustrated below is the relationship of wavelength and frequency of a transverse wave (Figure 2).
Figure 2. Definition of wavelength.

Wavelength: The distance from one crest of a wave to the next and is denoted by the Greek letter lambda \( \lambda \). Example in Figure 2 the wavelength is about 6.5 units.

Frequency: The number of crests, troughs, or any other point on the wave that passes a given point in a unit time interval. \( f \)

Amplitude: The maximum displacement of the wave from an equilibrium position. In the image above, the equilibrium position is 0 so the amplitude is 1. There is positive and negative displacement for each wavelength. The displacement is proportional to the amount of energy. Therefore, the greater the displacement, the larger the amount of energy associated with the wave.

Wavelength and frequency are inversely proportional, meaning that if the frequency goes up, the wavelength goes down. The same is true vice versa. As well, all forms of electromagnetic radiation travel at the same high velocity, the speed of light \( c \). The current accepted value for the speed of light is \( 2.99792458 \times 10^8 \) meters per second \( (\text{m/s}) \). For this lab the rounded value of \( 3.00 \times 10^8 \text{ m/s} \) is acceptable.

The relationship between frequency, wavelength, and the speed of electromagnetic radiation is given by:

\[
c = f \cdot \lambda
\]

Pre-Lab Questions:

1. Radio waves travel at the speed of light. A source emits radio waves with a wavelength of 6 cm. What is the frequency of the radio emission?

\[
\begin{align*}
3.0 \times 10^8 \text{ m/s} &= f (.06 \text{ m}) \\
5.0 \times 10^7 \text{ m/s} &= f
\end{align*}
\]

2. Radio waves travel at the speed of light. The Galileo spacecraft orbiting Jupiter sends a signal 670 million km to earth. How long does the signal take the reach earth? (velocity = distance/time)

\[
\begin{align*}
3.0 \times 10^8 \text{ m/s} &= 6.7 \times 10^{11} \text{ m/time} \\
2.23 \times 10^7 \text{ m} &= \text{time} \\
37.22 \text{ minutes} &= \text{time}
\end{align*}
\]
3. Compare the wavelength of a radio wave with a frequency of 1.42 GHz to the wavelength of a visible light wave with a frequency of $6 \times 10^{14}$ Hz.

\[
a) \ 3.0 \times 10^8 \text{m/s} = 1.42 \text{GHz} \times \lambda = 2.11 \times 10^{-2} \text{m} = 0.211 \text{m} \\
b) \ 3.0 \times 10^8 \text{m/s} = 6 \times 10^{14} \text{Hz} \times \lambda = 5.0 \times 10^{-7} \text{m}
\]

*The wavelength of the visible light wave is much shorter than the radio wave.*

4. Why can your radio pick up radio waves though walls, yet you cannot see through walls?

*The longer wavelengths of the radio waves can travel through solid objects. The shorter wavelength of visible or optical waves, however, cannot pass through solid objects.*

**Procedures**

1. Students should answer the pre-lab questions above before beginning the lab.

2. Instruct students to log into the Smiley Observation Control Room. For information on how to use Smiley, please refer to PARIPod Unit 2 Smiley basics.

3. With **Map** selected, click on the Sun as your source object. Click on **GO**.

4. Once the status bar reads “Mover stopped”, use **Hand Paddle** to position Smiley as close as you can to the source.

5. In **Continuum** mode choose the **Base Frequency** to be 1.42 GHz and your **IF GAIN** to be 10.

6. Now click on **Begin Scan**. Once the scan is complete and it starts to plot again, click on **Stop Scan**. To find the maximum intensity click on **Save Scan** and save your scan. Then click on **Open Data File** and click on **List** on the file you just saved. Look through the data points and find your maximum intensity. Record the maximum intensity in the data table below.

7. **Clear** the scan. While on the same object choose the **Base Frequency** to be 4.8 GHz.

8. Click on **Begin Scan** and follow the same procedure as you did in # 6 and 7. Record the maximum intensity in the data table.

9. Next, calculate the wavelength using the equation $c = f \lambda$ for each frequency and record it in the data table below.

10. Now take the ratio of the maximum intensity at 1.42 GHz to the maximum intensity at 4.8 GHz (divide the 1.42 GHz maximum signal by the 4.8 GHz maximum signal) and record it in the data table below.

11. Repeat steps 3-11 for two more objects above the horizon.
Data Collection

Ratio of Maximum Object Frequency Wavelength Maximum Signal Intensities

<table>
<thead>
<tr>
<th>Object</th>
<th>Intensities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun:</td>
<td>1.42 GHZ</td>
</tr>
<tr>
<td></td>
<td>4.8 GHZ</td>
</tr>
<tr>
<td>Object 2:</td>
<td>1.42 GHZ</td>
</tr>
<tr>
<td></td>
<td>4.8 GHZ</td>
</tr>
<tr>
<td>Object 3:</td>
<td>1.42 GHZ</td>
</tr>
<tr>
<td></td>
<td>4.8 GHZ</td>
</tr>
</tbody>
</table>

Analysis and Discussion

1. What can you observe from the ratios calculated in your results?

   The ratios are approximately 1.0. However, the 1.42 GHz signal is slightly stronger than the 4.8 GHz signal.

2. Using your data, explain how wavelength and frequency are inversely proportional.

   The wavelength for 1.42 GHz is $2.11 \times 10^1$ m while the wavelength for 4.8 GHz is $0.625 \times 10^1$ m. Therefore, the lower the frequency the longer the wavelength, and vice versa.

3. The sun is much more intense in the visible part of the electromagnetic spectrum than in the radio. This suggests that higher frequencies are more intense. Is this what you observed?

   No, the 1.42 GHz signal is more intense.
Unit 3 Lab 3

The Doppler Effect

Teacher Version
Unit 3 Lab 3: The Doppler Effect

Objective: In this lab, students will measure and compare the peak intensity of 21 cm radio waves emitted from the gas and dust between three different objects.

Teacher Notes:

**Note: It is recommended that you complete Lab 1 and Lab 2 before doing this lab.

PARIPod 3.3 is still in development.

Background Knowledge
Next time you are at a lake, watch the boats speed by. As they approach you the water waves are compressed in front of the boat and the waves behind the boat are spread out. In other words, as the boats approach, wavelengths of the water waves are short, but the wavelengths behind the boats are long.

![Figure 1. Boats generating waves.](image)

The same effect occurs with sound. Listen to a train go by. Blowing its whistle, the pitch is high as it approaches you. When it goes past, and away from you, the pitch is lower. Sound waves from the whistle are compressed as the train approaches and stretched out as the train recedes from you. (Figure 2 and Figure 3) You hear the short, compressed wavelengths as a higher frequency and the stretched, longer wavelengths as a lower frequency.
Like water and sound, light can be described as waves. The same principle of compressed and stretched wavelengths of light from moving light sources applies. This principle is called the Doppler Effect. The Doppler Effect applies to objects that produce any type of wave (sound, water, light). In terms of light emitted by a moving object, the Doppler Effect states that the observed frequency of emitted light from a moving object depends on the speed of the moving object. The equation for the Doppler Effect is:

\[ \frac{f_{\text{Dop}}}{f_{\text{Rest}}} = \frac{v}{c} \]
where $f_{\text{Dop}}$ is the Doppler shifted frequency, $f_{\text{Rest}}$ is the rest frequency, $v$ is the velocity of the source and $c$ is the speed of light ($3.00 \times 10^8$ m/s). After some algebra, the Doppler shift equation can be written as:

$$v = c \left( \frac{f_{\text{Dop}}}{f_{\text{Rest}}} \right)$$

Now we have an equation that we can use to find the speed of a source if we know the Doppler and the rest frequencies of the light emitted from the source.

Radio waves and visible light waves are all part of the electromagnetic spectrum. Radio waves just happen to have a longer wavelength and lower frequency than visible light. Thus an object emitting radio waves will also produce the Doppler Effect, if that object (or the observer) is moving. In fact, many astronomical objects emitting radio waves show the Doppler Effect. For example, supernova remnants, relatively dense interstellar clouds of neutral hydrogen, and quasars, starlike objects, all exhibit the Doppler Effect. If we measure the Doppler shifted and rest frequencies, then we can find the speed of the source.

**Procedures**

1. Login to Smiley. If you need any assistance with Smiley refer to to Unit 2: Smiley Basics.

2. In Map mode, pick one of the objects. We suggest Cygnus A, Virgo A, the Crab Nebula, or the Galactic Center depending on which of these are above the horizon. Each of these sources is emitting 1.42 GHz radio waves, but when these objects are moving their 1.42 GHz radio waves are shifted due to the Doppler Effect when they arrive at earth.

3. After you have picked an object click on GO.

4. Once Smiley has reached the target make sure Smiley is close to the target by comparing the TARGET Coordinates with the CURRENT Coordinates and using Hand Paddle.

5. Go into Spectrum mode to begin measuring intensity.

6. Set your PLOT RATE to 1x, your IF GAIN to around 19, your Base Frequency to 1.42 GHz and click on Begin Scan.

7. Watch the scan. The x-axis is the frequency and the y-axis is the intensity. The center of the x-axis is the 1.42 GHz frequency (right click on the graph and click on “show grid” to see the x and y axises). Lower frequencies are left of the center and higher frequencies are to the right of the center. Notice that the peak in intensity does not occur at exactly 1.42 GHz, because of the Doppler Effect!
8. Click on **Save Scan** and name the scan. Then click on **Open Data File**, find the scan you saved and list the data points.

9. Record the Doppler shifted frequency of your scan and calculate the velocity of the source in the table below.

10. Repeat procedures #2 – 8 for two more objects.

## Data Collection

<table>
<thead>
<tr>
<th>Object</th>
<th>Rest Frequency</th>
<th>Peak Intensity</th>
<th>Doppler shifted frequency</th>
<th>Velocity</th>
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Here is an example of a spectrum scan showing a Doppler shifted frequency from 1.42 GHz.

![Spectrum Scan Example](image)

The rest frequency is 1.42 GHz (1.42 \( \times 10^9 \) Hz) and the Doppler shifted frequency is 160 KHz (160 \( \times 10^3 \) Hz).
To calculate the Doppler shift from the above scan we use the equation:

\[ v = \frac{c(f_{\text{Dop}})}{f_{\text{Rest}}} \]

or

\[ v = (3.00 \times 10^8 \text{ m/s})\left[\frac{(160 \times 10^3 \text{ Hz})}{(1.42 \times 10^9 \text{ Hz})}\right] \]

Thus \( v = 3.38 \times 10^4 \text{ m/s} \)

**Analysis & Discussion**

1. Convert the velocity of the objects you observed from m/s to km/hr and compare them to the speed limit on a typical highway (100 km/hr).

   *Most of the objects will be moving at a much greater rate than the speed limit on a highway.*

2. Are the objects you observed moving towards or away from us? How do you know?

   *Answers depend on the objects observed. However, if the Doppler shifted frequency is positive, the object is moving towards us. If it is negative, the object is moving away from us.*
Unit 3 Lab 4

Mapping a Radio Source

Teacher Version
Unit 3 Lab 4:
Mapping a Radio Source

Objective: Students will use Smiley to map an astronomical radio source at a frequency of 1.42 GHz.

Teacher Notes:

**Note: It is recommended that you complete Lab 1 and Lab 2 before doing this lab.

PARIPod 3.3 is still in development.

Background Knowledge

Have you ever seen a contour map of a mountain like Mt. Everest shown here in Figure 1?

![Figure 1. Mount Everest.](image)
Figure 2 is a 3D image that shows a mountain like feature:

Figure 3 shows the same object from overhead:

Figure 4 shows the same map without the shading.
This is called a traditional contour map.

Pre-lab Activity
Print out (larger and try drawing contouring lines on the array of numbers. Start with the highest number and draw a circle around it. That will be the peak value. Then draw a closed curve around the next highest set of numbers for the next contour.

The contour lines show the peak and the valleys. In fact, the object above is actually the remnant of a supernova! The map was done at PARI at a frequency of 1.42 GHz. The map shows the intensity of the supernova remnant over an area on the sky of 1 degree x 1 degree larger than the moon! (The moon covers about 1/2 degree diameter on the sky).
The contour map in Figure 4 is nothing more than a grid of numbers, where numbers of the same value are connected together. The grid of numbers that make up the map in Figure 4 are:

\[
\begin{array}{ccccccccccc}
782 & 791 & 792 & 833 & 834 & 868 & 900 & 824 & 846 & 797 \\
782 & 830 & 799 & 942 & 853 & 944 & 873 & 849 & 826 & 797 \\
848 & 949 & 880 & 1180 & 961 & 1234 & 930 & 973 & 810 & 746 \\
957 & 1139 & 1103 & 1663 & 1286 & 1731 & 1121 & 1238 & 838 & 756 \\
1069 & 1363 & 1426 & 2173 & 1758 & 2430 & 1498 & 1567 & 932 & 799 \\
1177 & 1551 & 1825 & 2707 & 2415 & 2971 & 1505 & 1882 & 1045 & 890 \\
1202 & 1632 & 2163 & 2872 & 3029 & 3223 & 2449 & 2040 & 1250 & 933 \\
1148 & 1556 & 2277 & 2747 & 3291 & 3162 & 2701 & 1977 & 1308 & 920 \\
1018 & 1372 & 2170 & 2302 & 3185 & 2694 & 2632 & 1768 & 1310 & 880 \\
912 & 1109 & 1850 & 1736 & 2720 & 2007 & 2301 & 1470 & 1173 & 840 \\
\end{array}
\]

**Procedure:**

1. Login to Smiley. For information on how to use Smiley refer to Unit 2 Smiley Basics.

2. In Map mode either click on the Sun or select the Sun from the Control Panel Drop Down List. Now click on GO.

3. In Continuum mode set your IF GAIN to around 17 and make sure your Base Frequency is set to 1.42 GHz.

4. Click on Begin Scan. While scanning use Hand Paddle to find the peak intensity (Notice the box that displays the intensity for every point as it is plotted). In general the TARGET Coordinates should be close to the CURRENT Coordinates. Keep moving Smiley until you find the peak intensity. Record the peak intensity in the table below along with the CURRENT azimuth and altitude at which it occurs. Allow Smiley to keep scanning.

5. Fill in the top row and left-hand column of the table below with the azimuth – 6 degrees, 4 degrees, etc. Do the same for the altitude.

6. Mapping the Sun means that you need to measure the brightness across the disk of the Sun. Using the Hand Paddle set Smiley to 6 degrees of your peak intensity in both Altitude and Azimuth. Record the intensity in the table below.

7. Now move Smiley 4 degrees in the Azimuth from the peak intensity and keep the Altitude the same (6 degrees from peak intensity). Enter your results in the table below.

8. Next, move Smiley 2 degrees in the Azimuth from the peak intensity still keeping your Altitude the same. Enter your results in the table below.
9. Move Smiley +2 degrees in the Azimuth from the peak intensity still keeping your Altitude the same. Enter your results in the table below.

10. Move Smiley +4 degrees in the Azimuth from the peak intensity still keeping your Altitude the same. Enter your results in the table below.

11. Move Smiley +6 degrees in the Azimuth from the peak intensity still keeping your Altitude the same. Enter your results in the table below.

12. Continue moving Smiley using the **Hand Paddle** to adjust the altitude or azimuth and complete the table below.

13. From your set of measurements, construct a contour map.

14.

**Data Collection**

**Sun Intensities**

<table>
<thead>
<tr>
<th>Current Az at peak intensity=</th>
<th>Az – 6 degrees =</th>
<th>Az – 4 degrees =</th>
<th>Az – 2 degrees =</th>
<th>Az +2 degrees =</th>
<th>Az +4 degrees =</th>
<th>Az +6 degrees =</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Alt at peak intensity=</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Alt6 degrees=</td>
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<tr>
<td>Alt4 degrees=</td>
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<td>Alt2 degrees=</td>
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<td>Alt+2 degrees=</td>
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<td>Alt+4 degrees=</td>
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<tr>
<td>Alt+6 degrees=</td>
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</tbody>
</table>
Analysis & Discussion

1. Does your map show a central peak and some structure? Why is the central peak off center?

   Yes, there should be a peak with some decreasing intensities surrounding it. It is difficult to record the radio waves simultaneously from every point on the disk of the sun. Therefore, the central peak may be off center because the Earth is moving constantly, thus the measurements of radio waves coming from the sun are going to shift with the Earth.

2. What is the area on the sky, in degrees, that you have mapped?

   12 x 12

2. Will every object have the same structure? What might the map look like if you used more points more closely spaced?

   No, each object will emit different intensities of radio waves. If you used more points more closely spaced, the differences in intensities would be less. The map would have more closely spaced lines.
Pre-Lab Activity (Large Version)
Print out (larger and try drawing contouring lines on the array of numbers.
Start with the highest number and draw a circle around it. That will be the peak value.
Then draw a closed curve around the next highest set of numbers for the next contour.

The contour lines show the peak and the valleys. In fact, the object above is actually the
remnant of a supernova! The map was done at PARI at a frequency of 1.42 GHz. The
map shows the intensity of the supernova remnant over an area on the sky of 1 degree x 1
degree larger than the moon! (The moon covers about 1/2 degree diameter on the sky).

782  791  792  833  834  868  900  824  846  797
782  830  799  942  853  944  873  849  826  797
848  949  880  1180  961  1234  930  973  810  746
957  1139  1103  1663  1286  1731  1121  1238  838  756
1069  1363  1426  2173  1758  2430  1498  1567  932  799
1177  1551  1825  2707  2415  2971  1505  1882  1045  890
1202  1632  2163  2872  3029  3223  2449  2040  1250  933
1148  1556  2277  2747  3291  3162  2701  1977  1308  920
1018  1372  2170  2302  3185  2694  2632  1768  1310  880
912  1109  1850  1736  2720  2007  2301  1470  1173  840