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| **Title** | Man vs. Beast: Approximating Derivatives using Human and Animal Movement |
| **Introduction** | In this lesson, students will develop the ability to determine numeric approximations of instantaneous rate of change. Students will develop this understanding in the context of analyzing the velocity and acceleration of human and animal movement.  Connection to Research: I worked with Dr. Laura Miller and Dr. Michael Minion of the University of North Carolina at Chapel Hill. Both Dr. Miller and Dr. Minion work in fluids dynamics research. Specifically, Dr. Miller uses video analysis software to study how organisms interact with fluids. This project is designed to bring her research of mathematics and biology into the Calculus classroom. Specifically, students are using similar video analysis software to analyze the motion of organisms. |
| **Curriculum Alignment** | **These are the standards from the College Board AP Calculus AB Syllabus**  Use data to approximate |
| **Learning Outcomes** | By the end of this project, students, when given a table of values representing position vs. time, will be able to accurately approximate an object’s velocity and acceleration. Additionally, students will be able to make the connection between the numeric approximation of instantaneous velocity and acceleration and a function’s first and second derivatives. |
| **Time Required and Location** | Two, 90 minute periods |
| **Materials Needed** | * Computer * Ruler * Logger Pro 3 software * Access to Animal Videos (see attached videos and Supplemental Information Section for website links) * Use of a digital video camera (optional)   **Technology Resources**   * Computer * Ruler * Logger Pro 3 software * Access to Animal Videos (see websites and video attachments) * Use of a digital video camera (optional) |
| **Participant Prior Knowledge** | Before beginning this lesson, students should be familiar with the process of finding a derivative and sketching the graph of a derivative, given a function. Additionally, students will understand that a derivative represents the slope of a line tangent to a curve and that the derivative of a position function is velocity and the derivative of velocity is acceleration. |
| **Activities** | **Day One:**   * 1. *Warm-Up***:** As they enter class students will be given a card containing either a graph or a description of a graph. Each card makes up one piece of a four card sequence (original function, 1st derivative, 2nd derivative, description of the original function). Students will then work as a class to find the other pieces to complete sequence (This is not my activity and I am not sure how to cite the teacher that created it).   2. *Activity***:**  1. Students were placed in groups of three at the start of the project and they will continue working with those groups during this lesson. If this lesson is being used outside of the complete project, students should still be placed in groups of three. 2. Using digital cameras, students will collect video of a person dropping a ball and will upload this video onto Logger Pro 3 (see Supplemental Information section for instructions on using Logger Pro 3). If you do not have access to digital cameras, I have included links to a ball dropping video that can be used with Logger Pro 3 that will achieve the same results. 3. Using the ball drop video, students will sketch a graph of the ball’s position vs. time using the video analysis feature of Logger Pro. The software will allow students to track the position of the ball frame-by-frame and will produce both a graph of position vs. time and a data table for position vs. time. Using the data table, students will create a graph, on paper, of the ball’s position vs. time. 4. Using a ruler, students will approximate the slopes of lines connecting the data from each frame. Not all students will immediately see this method of approximating the derivative and the teacher will need to model the approach. This process will allow students to create a table representing the ball’s velocity vs. time. 5. As this is part of a larger project aimed at analyzing human and animal movement, students will then strategize movements for which they would like to compare velocity and acceleration with the movements of specific animals. For example, students may want to compare their acceleration at the start of a jump with that of a frog, or their sprinting speed with the speed of a lizard. After determining which human movements they would like to analyze, students will collect video of those movements and analyze them using Logger Pro 3. Student analysis will include graphs of position vs. time, velocity vs. time, and acceleration vs. time, as well as data tables for position vs. time, velocity vs. time, and acceleration vs. time. Thus, this lesson links nicely with the curve sketching lessons of the previous days. 6. *Lesson Wrap-Up:* During this phase of the lesson (the last 10-15 minutes of class), the teacher should ask students questions to guide students toward the connections that are needed from this lesson. Here are some questions that can be used as a guide:    1. *What were some strategies you used to determine the slopes of tangent lines?*    2. *How were you able to determine where to position your ruler to determine the approximate slope of the tangent line?*    3. *Take two minutes to write a general rule for approximating a derivative given a table of values. After two minutes, allow different students to share their findings.* 7. *Exit Ticket:* As an exit ticket, students will be given a table of values representing an object’s position vs. time and will be asked to approximate the values for velocity vs. time (solution attached).  |  |  | | --- | --- | | Time (s) | Position (m) | | 0 | 2.35 | | 2 | 2.67 | | 5 | 2.82 | | 6 | 3.11 |   **Day Two:**   * + 1. *Warm-Up:* Students will be asked to write a journal describing the goal of the previous day’s activities and outlining the goals for the day (what movements will be analyzed? What animal videos need to be collected? Etc.).     2. *Activity:*   Students will collect two more human movements and two more animal movements to use for the comparison study. This activity will be mostly self-driven for the students as they work to complete the project. During the class, the teacher should offer workshops to re-teach important concepts with which students may be struggling. A workshop is 10 – 15 minutes of direct instruction that serves as remediation. Workshop topics are normally student selected, but they can be predicted. Students will most likely need workshops on Curve Sketching and Numerical Approximations of the derivative. As the class works, small groups of students attend these workshops.  Students will use the remainder of class to continue analyzing the human and animal movements and creating a pamphlet on the Man vs. Beast analysis. |
| **Assessment** | Warm-Ups  Position vs. Time Data Table at the end of Day One – Students will be given a position vs. time data table and will be asked to approximate the velocity vs. time data table.  Final Pamphlet for Man vs. Beast |
| **Critical Vocabulary** | * Position – the net distance traveled by an object * Acceleration – the rate of change of an object’s velocity * Velocity – the rate of change of an object’s position * Speed – the absolute value of an object’s velocity * Derivative – the slope of the line tangent to a curve at a given point * Critical Points – points on a function including extrema, inflection points, and zeros * Local Maximum/Minimum – the point on a function where the 1st derivative is 0 and the points surrounding that point are positive before the point and negative after or vice versa. |
| **Modifications** | Workshops for remediation  Computer graphing software for students with motor impairments |
| **References** | Websites for animal videos:  Bruce C. Jayne, University of Cincinnati, Videos of Locomotion & Behavior  http://www.artsci.uc.edu/collegedepts/biology/fac\_staff/jayne/videos.aspx |
| **Supplemental Information** | Websites for animal videos:   * http://www.artsci.uc.edu/collegedepts/biology/fac\_staff/jayne/videos.aspx * http://www3.science.tamu.edu/cmse/videoanalysis/ * I will have students collect much of the human movement and ball dropping video using Digital Flip Cameras, as this will increase student investment in the analysis.   Instructions for using Logger Pro 3 to create position vs. time graphs from actual video footage:  **Directions for using the Video Analysis Feature of Logger Pro 3**   1. Open Logger Pro 2. Select “Insert,” then “Movie” 3. Double-click the movie you would like to analyze 4. Move the movie to the center of the screen by clicking and dragging 5. In the bottom right corner of the movie, click on the three red dots leading to a black triangle 6. In the menu that appears, click the third button from the top (“Set Origin”) 7. Click the point in the movie that you would like to be the origin 8. In the same menu, click the fourth button from the top (“Set Scale”). This will be a yellow ruler. 9. Click on a spot and hold to draw a line on the screen. This should be a known length (for instance, my green line traces a meter stick in the movie, so I know it is 1 meter). Enter the length of the green line in the movie. 10. In the same menu, click the second button from the top (“Add Point”) 11. Find a spot on the object that you can easily follow throughout the movie (for me it is the center of the ball I am following). Place a point on this spot by left clicking. 12. The movie will move forward one frame. Find the same spot and place another point. Continue this process until you have finished the portion of the movie you would like to analyze. 13. When you are done with the movie, a graph will appear behind the movie. Delete the Movie to look at your graph. The blue points represent the y-coordinates vs. time and the red points represent the x-coordinates vs. time. 14. The data table will appear on the left of the Logger Pro Window. Now you can analyze the data further. |
| **Author Info** | My name is Michael Belcher. I teach Algebra 1 and AP Calculus AB at Hillside New Tech High School in Durham, NC. We are a wall to wall project based learning school. Our goal is to foster academic success through real world applications of the Standard Course of Study. I am currently in my 3rd year of teaching at Hillside New Tech High School and my 6th year of teaching overall. I graduated with a B.A. in Mathematics and minors in Physics and Secondary Education from Wake Forest University in Winston-Salem, NC. I earned an M.A. from Teachers College Columbia University in New York, NY. |

**Solutions to Exit Ticket**

1. *Exit Ticket:* As an exit ticket, students will be given a table of values representing an object’s position vs. time and will be asked to approximate the values for velocity vs. time (solution attached).

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| Time (s) | Position (m) | Approximate Velocity (m/s) |
| 0 | 2.35 |  |
| 2 | 2.67 | 0.118 |
| 5 | 2.82 | 0.11 |
| 6 | 3.11 |  |