**Title**

Weight and Balance of an Airplane

**Introduction**

This series of lessons focuses on exposing high school math students to relevant applications in the aviation industry. This is the first in the series for the Integrated Math 3 level. This can be taught in conjunction with Unit 2 Lesson 2 – Aviation Performance but can also be a stand-alone lesson. In this lesson, students will explore the concept of the weight and balance of an airplane to determine whether or not a plane could successfully fly.

**Learning Outcomes**

 Students will calculate the Weight and Balance of an airplane. Students will articulate the importance of correctly calculating the weight and balance of a plane and the ramifications of incorrectly calculating it. Students will explain how this skill is used in an aviation career.

**Curriculum Alignment**

This lesson falls under two of the Mathematical Practices laid out in the Common Core Standards.

Standard 1: Make sense of problems and persevere in solving them.

Standard 2: Reason abstractly and quantitatively.

It directly applies to the Quantities Domain (N-Q):

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

**Classroom Time Required**

1-1.5 90 minute block periods or 2-3 45 minute class periods.

**Teacher Preparation**

* Test out the paper airlines activity.
* Make sure your email address can handle all of the large files that the students are going to send you. Set up a free account with Gmail or Hotmail if you need it.
* Work through the problems so you understand the steps. This is not something you want to try for the first time in front of the kids.

**Materials Needed**

* 3-4 paperclips for each group
* Blank white paper
* Tape measure (optional)
* Weight and Balance 1 Worksheet
* Weight and Balance 2 Worksheet
* Cessna 152 and 172 information

**Technology Resources**

* cell phone camera or digital camera
* email address capable of receiving images by email
* calculators
* computer, projector and speakers for showing the digital video

**Pre-Activities for Students**

None

**Activities**

Part 1:

1. Split students up into pairs and go outside or find a long hallway. The gym or cafeteria would be a good option as well for a rainy or cold day.

2. Give each group blank paper, paperclips (5 or 6), and a tape measure (optional).

3. Have them build their favorite type of airplane and launch it. Record their observations on the Paper Airplane Data Sheet.

4. Have them place first one paperclip, then multiples, in various locations on their airplane and relaunch the planes. Record their observations.

5. Challenge the students to find the optimal arrangement of 3 or 4 paperclips to allow the plane to fly the farthest.

6. Have the students take a picture of their plane with their cell phone or digital camera and email it to you.

7. You may want to turn this into a competition and offer a prize for the group whose plane goes the furthest. If you do, you may want to turn it into a design competition.

Part 2:

 Back in the classroom:

* “What happened when you put all the paperclips on the front of your plane?”
* “This is called being nose heavy. If an actual plane has all of its weight up front, it will crash nose-first.”
* “What happened when you put all the paperclips at the back of your plane?”
* “This is called being tail heavy. If an actual plane has all of its weight at the back of the plane it will go into a stall and crash.”
* “Where was the best arrangement of paperclips?”
* “Let’s talk about real planes. When they load bags onto an airplane, do they just toss them in there or do they anchor them down?”
* “Have you ever been on a mostly empty plane and a flight attendant asked you to move seats?”
* “They do that so the weight of the plane will be properly balanced and won’t crash.”
* “There are actual calculations that each pilot is supposed to do every time before they take off. They need to find the center of gravity of the loaded plane and make sure it falls into a specified range or the plane will not fly correctly. These calculations are called finding the weight and balance of a plane.”

Go through the critical vocabulary and have them label their plane with the terms. Keep the airplane out and accessible during the rest of the lesson so they have a concrete example to refer back to.

Part 3:

 Lead them through the first example of the Weight and Balance Worksheet and then let them work through the rest of it in their groups. Circulate through the room trouble-shooting and keeping them on track.

Part 4: (Two options)

1. There is a National Geographic series called “Air Crash Investigations”. One episode deals with a plane crash in Charlotte in 2003 where a pilot had incorrect information to calculate his CG envelope and consequently, the plane was not correctly balanced and crashed shortly after takeoff. The following links take you to the five parts of the episode.

Season 5 – Dead Weight Episode 5

http://www.youtube.com/watch?v=f5x5F94iiB0 – part 1

http://www.youtube.com/watch?v=bHdXP0JfbZE&feature=related – part 2

http://www.youtube.com/watch?v=\_kxekbpWgfU&feature=related – part 3

http://www.youtube.com/watch?v=K2zY7YDk8CA&feature=related – part 4

http://www.youtube.com/watch?v=5XsmEAvFIgU&feature=related – part 5

2. If you don’t have enough time to watch the Air Crash Investigations episode, you could discuss the background information of the crash and then show the NTSB recreation of the flight.

http://www.youtube.com/watch?v=UllYdX5Nk1E – NTSB recreation, Charlotte USAir flight

**Guided Practice**

Have them complete the Weight and Balance 2 worksheet on their own in class or for homework. Discuss at the end of class or the next day to check for understanding.

**Assessment**

Unit 2 Quiz or have them complete the Weight and Balance 2 worksheet instead.

**Modifications**

* Add in a competition for the farthest plane flight.
* Watch either the NTSB recreation (short) or the Air Crash Investigations episode (long) about Flight 5481.

**Alternative Assessments**

They can do the quiz found with the Unit 2 Lesson 2 plan or do the Weight and Balance 2 worksheet instead.

**Critical Vocabulary**

* Basic Empty weight: the weight of the plane with all of its equipment but no fuel or payload.
* Payload: passengers, crew, baggage, and cargo
* Reference Datum: a random vertical line in the plane that all calculations are based on.
* Arm: the distance from the datum to a weight in the plane (forward of the datum is negative and aft of the datum is positive)
* Moment: caused by a weight on the end of an arm. Calculated by multiplying the weight of an object times the length of the arm (units are pounds-inches)
* CG arm: the length from the center of gravity to the datum. Calculated by dividing the sum of the weights by the sum of the moments.

**Websites and Resources**

* Wikipedia article on the Center of Gravity of an Airplane for background information. <http://en.wikipedia.org/wiki/Center_of_gravity_of_an_aircraft>
* See Activity Section, part 4
* A website with directions for building different types of paper airplanes. <http://www.10paperairplanes.com/>
* [www.zamzar.com](http://www.zamzar.com) will convert videos into a file that can be downloaded and saved onto your computer.

**Comments**

* When you do the paper airplane activity, make sure you are somewhere you can spread out. You may want to consider eye protection. Seriously. Trust me on this.
* This is the first of two lessons in the Integrated Math 3 Unit. One unit precedes this one, for Integrated Math 2, and one follows, for Integrated Math 4, which also deal with aviation applications.

**Author Information**

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 Audrey Floyd is the Department Chair of the Aviation Management / Career Pilot Technology Program at Guilford Technical Community College. She was an Air Force pilot with over 2600 hours of flight time. Audrey was also a high school physics teacher in Davidson County before she began at Guilford Tech.

**Paper Airplane Data Sheet**

|  |  |
| --- | --- |
| **Sketch** | **Flight Length** |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Optimal Arrangement:

**Weight and Balance of an Airplane**

Hints:

* If the plane is overweight, it won’t fly. Check the total weight before you do the rest of the calculations so you don’t waste your time.
* Aviation gasoline weighs 6 lbs per gallon. Make sure you convert gallons to pounds before you plug it into your table.

**Examples: (Refer to your info about the Cessna 152 for arms, maximum weight limits, CG moment envelope, etc)**

**1.**  Basic Empty Weight is 1194 lbs with a moment of 40,000 lb-in

 The pilot weighs 180 lbs

The plane has 24.5 gal of fuel.

Baggage area #1 has 40 lbs in it and baggage area #2 has 10 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** |  |  |  |
| **Pilot** |  |  |  |
| **Fuel** |  |  |  |
| **BA #1** |  |  |  |
| **BA #2** |  |  |  |
| **Total** |  | n/a |  |

Is the plane overweight? \_\_\_\_\_\_\_\_\_\_\_\_

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 lb

Is the CG arm within the CG limits? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**2.**  Basic Empty Weight is 1194 lbs with a moment of 40,000 lb-in

 The pilot and passenger weigh 380 lbs

The plane has 24.5 gal of fuel.

Baggage area #1 has 40 lbs in it and baggage area #2 has 10 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** |  |  |  |
| **Pilot** |  |  |  |
| **Fuel** |  |  |  |
| **BA #1** |  |  |  |
| **BA #2** |  |  |  |
| **Total** |  | n/a |  |

Is the plane overweight? \_\_\_\_\_\_\_\_\_\_\_\_

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 lb

Is the CG arm within the CG limits? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**3.** Basic Empty Weight is 1194 lbs with a moment of 40,000 lb-in

 The pilot weighs 180 lbs

The plane has 13 gal of fuel.

Baggage area #1 has 120 lbs in it and baggage area #2 has 40 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** |  |  |  |
| **Pilot** |  |  |  |
| **Fuel** |  |  |  |
| **BA #1** |  |  |  |
| **BA #2** |  |  |  |
| **Total** |  | n/a |  |

Is the plane overweight? \_\_\_\_\_\_\_\_\_\_\_\_

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 lb

Is the CG arm within the CG limits? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Examples: (Refer to your info about the Cessna 172 for arms, maximum weight limits, CG moment envelope, etc)**

**4.** Basic Empty Weight is 1660 lbs with a moment of 66,000 lb-in

 The pilot and front seat passenger weighs 360 lbs and are of average height

The rear passenger weighs 180 lbs and is also average height.

The plane has 31 gal of fuel.

Baggage area #1 has 70 lbs in it and baggage area #2 has 30 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** |  |  |  |
| **Pilot & FP** |  |  |  |
| **RP** |  |  |  |
| **Fuel** |  |  |  |
| **BA #1** |  |  |  |
| **BA #2** |  |  |  |
| **Total** |  | n/a |  |

Is the plane overweight? \_\_\_\_\_\_\_\_\_\_\_\_

How much fuel can the plane carry? \_\_\_\_\_\_\_\_\_\_\_\_

Redo your calculations with the new fuel amount.

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 lb

Is the CG arm within the CG limits? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**5.** In the situation above, the pilot and front seat passenger are both very tall and have to slide their seats all the way back to sit comfortably. How does that affect the CG arm?

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** |  |  |  |
| **Pilot & FP** |  |  |  |
| **RP** |  |  |  |
| **Fuel** |  |  |  |
| **BA #1** |  |  |  |
| **BA #2** |  |  |  |
| **Total** |  | n/a |  |

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 lb

**Weight and Balance of an Airplane – Answer Key**

Hints:

* If the plane is overweight, it won’t fly. Check the total weight before you do the rest of the calculations so you don’t waste your time.
* Aviation gasoline weighs 6 lbs per gallon. Make sure you convert gallons to pounds before you plug it into your table.

**Examples: (Refer to your info about the Cessna 152 for arms, maximum weight limits, CG moment envelope, etc)**

**1.**  Basic Empty Weight is 1194 lbs with a moment of 40,000 lb-in

 The pilot weighs 180 lbs

The plane has 24.5 gal of fuel.

Baggage area #1 has 40 lbs in it and baggage area #2 has 10 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** | 1194 |  | 40,000 |
| **Pilot** | 180 | 39 | 7020 |
| **Fuel** | 147 | 42 | 6200 |
| **BA #1** | 40 | 64 | 2560 |
| **BA #2** | 10 | 84 | 840 |
| **Total** | 1572 | n/a | 56, 620 |

Is the plane overweight? \_\_NO\_\_\_\_\_\_\_\_\_\_

CG arm= \_\_56,620\_lb-in\_ = 36.02 in

 1572 lb

Is the CG arm within the CG limits? \_Yes\_\_\_\_\_\_\_\_\_\_\_\_\_

**2.**  Basic Empty Weight is 1194 lbs with a moment of 40,000 lb-in

 The pilot and passenger weigh 380 lbs

The plane has 24.5 gal of fuel.

Baggage area #1 has 40 lbs in it and baggage area #2 has 10 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** | 1194 |  | 40,000 |
| **Pilot** | 380 | 39 | 14,820 |
| **Fuel** | 147 | 42 | 6200 |
| **BA #1** | 40 | 64 | 2560 |
| **BA #2** | 10 | 84 | 840 |
| **Total** |  | n/a |  |

Is the plane overweight? \_\_YES\_\_\_\_\_\_\_

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 lb

Is the CG arm within the CG limits? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**3.** Basic Empty Weight is 1194 lbs with a moment of 40,000 lb-in

 The pilot weighs 180 lbs

The plane has 13 gal of fuel.

Baggage area #1 has 120 lbs in it and baggage area #2 has 40 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** | 1194 |  | 40,000 |
| **Pilot** | 180 | 39 | 7,020 |
| **Fuel** | 78 | 42 | 3,276 |
| **BA #1** | 120 | 64 | 7,680 |
| **BA #2** | 40 | 84 | 3,360 |
| **Total** | 1612 | n/a | 61,336 |

Is the plane overweight? \_NO\_\_\_\_\_\_\_

CG arm= \_\_\_61,336 lb-in\_ = 38.0 in

 1612 lb

Is the CG arm within the CG limits? \_NO\_\_\_\_\_

**Examples: (Refer to your info about the Cessna 172 for arms, maximum weight limits, CG moment envelope, etc)**

**4.** Basic Empty Weight is 1660 lbs with a moment of 66,000 lb-in

 The pilot and front seat passenger weighs 360 lbs and are of average height

The rear passenger weighs 180 lbs and is also average height.

The plane has 31 gal of fuel.

Baggage area #1 has 70 lbs in it and baggage area #2 has 30 lbs in it.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** | 1660 |  | 66,000 |
| **Pilot & FP** | 360 | 48 | 13,320 |
| **RP** | 180 | 73 | 13,140 |
| **Fuel** | 186 (150) | 48 | 8928 (7,200) |
| **BA #1** | 70 | 95 | 6,650 |
| **BA #2** | 30 | 115 | 3,450 |
| **Total** | 2486 (2450) | n/a | 111,488 (109,760) |

Is the plane overweight? \_\_YES\_\_\_\_\_\_\_

How much fuel can the plane carry? \_\_25 gal\_\_\_\_

Redo your calculations with the new fuel amount. (in parenthesis)

CG arm= \_109760 lb-in\_ = 44.8 in

 2450 lb

Is the CG arm within the CG limits? \_\_\_YES\_\_\_\_\_\_\_\_\_\_

**5.** In the situation above, the pilot and front seat passenger are both very tall and have to slide their seats all the way back to sit comfortably. How does that affect the CG arm?

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** | 1660 |  | 66,0000 |
| **Pilot & FP** | 360 | 46 | 16560 |
| **RP** | 180 | 73 | 13,140 |
| **Fuel** | 150 | 48 | 7,200 |
| **BA #1** | 70 | 95 | 6,650 |
| **BA #2** | 30 | 115 | 3,450 |
| **Total** | 2450 | n/a | 113,000 |

CG arm= \_\_113,000 lb-in\_ = 46.1 in

 2450 lb

**Weight and Balance 2**

Use the Cessna 172 data for these situations.

The following two situations have the same total weight, yet their CG arms are almost 3 inches different.

**Situation #1**

The Basic Empty Weight is 1660 lbs with a moment of 66,000 lb-in.

Pilot and front passenger together weigh 360 lbs and are short so their seats are in the front position.

The plane has a full load of fuel, 31 gallons.

Baggage area #1 has 140 lbs of cargo and baggage area #2 has 100lbs.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** |  |  |  |
| **Pilot & FP** |  |  |  |
| **RP** |  |  |  |
| **Fuel** |  |  |  |
| **BA #1** |  |  |  |
| **BA #2** |  |  |  |
| **Total** |  | n/a |  |

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 Lb

**Situation #2**

The Basic Empty Weight is 1660 lbs with a moment of 66,000 lb-in.

Pilot and front passenger together weigh 360 lbs and are short so their seats are in the front position.

The plane has a full load of fuel, 31 gallons.

The rear passenger weighs 240 lbs and there is no cargo.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** |  |  |  |
| **Pilot & FP** |  |  |  |
| **RP** |  |  |  |
| **Fuel** |  |  |  |
| **BA #1** |  |  |  |
| **BA #2** |  |  |  |
| **Total** |  | n/a |  |

CG arm= \_\_\_\_\_\_\_\_\_\_lb-in\_ = in

 Lb

Why do these situations have the same weight but different CG arms?

Is it simply enough to know whether your plane is under the maximum weight or does where those weights are located make a difference?

**Weight and Balance 2 – Answer Key**

Use the Cessna 172 data for these situations.

The following two situations have the same total weight, yet their CG arms are almost 3 inches different.

**Situation #1**

The Basic Empty Weight is 1660 lbs with a moment of 66,000 lb-in.

Pilot and front passenger together weigh 360 lbs and are short so their seats are in the front position.

The plane has a full load of fuel, 31 gallons.

Baggage area #1 has 140 lbs of cargo and baggage area #2 has 100lbs.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** | 1660 |  | 66,000 |
| **Pilot & FP** | 360 | 34 | 12,240 |
| **RP** | 0 |  | 0 |
| **Fuel** | 186 | 48 | 8,920 |
| **BA #1** | 140 | 95 | 13,300 |
| **BA #2** | 100 | 115 | 11,500 |
| **Total** | 2446 | n/a | 111,968 |

CG arm= \_\_111,968 lb-in\_ = 45.78 in

 2446 lb

**Situation #2**

The Basic Empty Weight is 1660 lbs with a moment of 66,000 lb-in.

Pilot and front passenger together weigh 360 lbs and are short so their seats are in the front position.

The plane has a full load of fuel, 31 gallons.

The rear passenger weighs 240 lbs and there is no cargo.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Weight (lbs)** | **Arm (in)** | **Moment (lb-in)** |
| **BEW** | 1660 |  | 66,000 |
| **Pilot & FP** | 360 | 34 | 12,240 |
| **RP** | 240 | 73 | 17,520 |
| **Fuel** | 186 | 48 | 8,920 |
| **BA #1** | 0 |  | 0 |
| **BA #2** | 0 |  | 0 |
| **Total** | 2446 | n/a | 104,688 |

CG arm= \_104,688 lb-in\_ = 42.80 in

 2446 lb

Why do these situations have the same weight but different CG arms?

The weight is distributed differently in the two situations.

Is it simply enough to know whether your plane is under the maximum weight or does where those weights are located make a difference?

Where the weights are located makes a huge different and can determine whether or not the plane is safe to fly.