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| **Title**  | Using Motion Sensors to Explore Graph Sketching |
| **Introduction**  | This lesson is designed to allow students an opportunity to explore the relationship between the graph of a function and the graphs of its first and second derivatives. Students will also examine the concept of differentiability through a bodily-kinesthetic approach. Connection to Research: I worked with Dr. Laura Miller and Dr. Michael Minion of the University of North Carolina at Chapel Hill. Both Dr. Miller and Dr. Minion work in fluids dynamics research. Specifically, Dr. Miller uses video analysis software to study how organisms interact with fluids. This project is designed to bring her research of mathematics and biology into the Calculus classroom. Specifically, students are using similar video analysis software to analyze the motion of organisms. |
| **Curriculum Alignment**  | **These are the standards from the College Board AP Calculus AB Syllabus**Understanding the relationship between the graphs of $f, f^{'}, f''$Using the graph of a function to sketch a graph of its first and second derivatives |
| **Learning Outcomes**  | By the end of this lesson, students, when given the graph of a function, will be able to accurately sketch the graph of the function’s first and second derivatives. Students will be able to explain the relationship between position, velocity, and acceleration. Additionally, students will be able create their own position vs. time graph and sketch the graph for velocity and acceleration vs. time for real world circumstances. |
| **Time Required and Location**  | One 90-minute period |
| **Materials Needed**  | * + CB-L motion sensors
	+ Ti-84 Calculators
	+ Videos for assessment (attached)
	+ Graphing Worksheet (attached)

**Technology Resources*** + CB-L motion sensors
	+ Ti-84 Calculators
	+ Videos for assessment (attached)
	+ A computer projector or a TV (to show the videos for the assessment phase of the lesson)
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| **Participant Prior Knowledge**  | Before beginning this lesson, students should have experience with finding limits and using the formal definition of a derivative. Students will be able to find the derivative of a function and will be able to explain the meaning of the derivative, as it relates to slopes of tangent lines, and the information a derivative provides about the graph of the function. For instance, students should know that when the derivative is negative, the tangent line has a negative slope, which means the function is decreasing. Students should know that a negative velocity refers to the direction of movement. |
| **Activities**  | **Warm-Up (5-10 minutes):** This is optional. My classes always begin with a short 5-10 minute warm-up that introduces the focus of the day’s lesson by reviewing previously learned material. In this warm-up, students will be asked to graph three functions and find the derivatives of each function at specified points (worksheet attached).**Motion Sensing Activity (45 minutes):**Place students in groups of three. If possible, group students according to the following readiness levels:* + - 1. Tier 1: Students have not yet mastered the graphs of parent functions (for instance, these students would have difficulty graphing the natural log function or piecewise functions).
			2. Tier 2: Students have a general idea of the shape and behavior of most of the parent functions, but they still rely heavily on the calculator and are unsure about specific points (for instance, these students know that a sine curve oscillates, but do not consistently see that the graph oscillates between -1 and 1 or that the period is 2π.
			3. Tier 3: Students have mastered the graphs of parent functions and can consistently sketch accurate graphs of most functions without a calculator.

Within each group, students must assign three group roles (5 minutes):1. Data Collector – This student will be responsible for holding the motion sensor and Ti-84 Calculator. The data collector must start and stop the data collection as well as clearly communicate time intervals for critical points (when should the walker stop, turn around, etc.).
2. Observer/Recorder – This student will be responsible for providing a written log of the walker’s movements. It will be important for the Observer/Recorder to accurately describe not just the direction of the Walker, but also the speed and acceleration of the walker (this can be done in common language – speeding up, slowing down, walking the same speed, etc.).
3. Walker – This student will be responsible for physically creating the different graphs (see worksheet). He/she will be in charge of walking in front of the motion sensor and providing the data for the data collector.

Once roles have been assigned, students will be given their target data.1. Tier 1 groups: Five functions and their graphs
2. Tier 2 groups: Five functions without graphs – They can use their calculators
3. Tier 3 groups: Five functions – no graphs, no calculators

Before beginning the process of creating the target data, students should be given time (3-4 minutes) to experiment with the motion sensing devices. As a class, have students attempt to create the graph of $y=x\left(x-4\right)$ with the motion sensors (be sure to draw or project the graph for all groups to see. The goal of this activity is to learn about the motion sensors)Students will then be given time (10 minutes) to devise a plan for creating the target data. Groups must examine the position vs. time graphs and construct a script for the walkers to follow. For instance, if the group is trying to create the graph of $y=x\left(x-4\right)$, they would want the walker to start very close to the motion sensor, then walk backwards for 2 seconds, quickly at first, but slowing down as you get to the 2 second mark. After 2 seconds, the walker would need to slowly start to return to the sensor, speeding up until he/she has returned to the sensor. These scripts do not have to be right at first. Students should be allowed to return to their scripts and alter them as needed.Once all students have completed their scripts, they will be given time to create their target data (30 minutes). Students should rotate roles for every function. Be sure that each group member is fulfilling his/her role. It will be essential that the observer is recording everything that the Walker is doing to create the target data.Students will now sketch the graph of the velocity (first derivative) of the walker and the acceleration (second derivative) of the walker by referring the notes collected by the observer/recorder.**Activity Wrap Up (10 minutes):**The teacher should discuss the findings from the motion sensor exploration.* + - 1. How did you walk, in terms of speed, when you were creating the first graph (a straight line that is increasing)?
			2. Was anyone able to create the natural log graph? How?
			3. Were there any graphs that you were unable to create? Why?
			4. What was difficult about creating the absolute value function? Do you think it would be possible for a person to create that graph?
			5. When a graph was increasing, what did you notice about the graph of the velocity? What did you notice when the graph was decreasing?
			6. When you created the graph of the quadratic function, what did you notice about the velocity of the walker at the maximum value?
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| **Assessment**  | 1. **Assessment (20 minutes):**

Students will be shown a picture of an animal (this will be the opening shot of a video) and asked to sketch a graph of the animal’s position vs. time graph for the motion that will follow (here students need to know that this is a movie and that they are predicting what happens next, otherwise this will not make sense). Students will then be asked to sketch a velocity vs. time graph and acceleration vs. time graph for the animal’s movement. Next, students will discuss their graphs with the small groups, explaining why their graphs look the way they do, based on the movement they expect to see. After students have been given an opportunity to discuss their predictions, show the class the video of the animal movement. Allow students time to create a new graph for position vs. time, velocity vs. time, and acceleration vs. time. Repeat the activity with a 2nd video if time permits. |
| **Critical Vocabulary**  | * Position – the net distance traveled by an object
* Acceleration – the rate of change of an object’s velocity
* Velocity – the rate of change of an object’s position
* Speed – the absolute value of an object’s velocity
* Derivative – the slope of the line tangent to a curve at a given point
* Critical Points – points on a function including extrema, inflection points, and zeros
* Local Maximum/Minimum – the point on a function where the 1st derivative is 0 and the points surrounding that point are positive before the point and negative after or vice versa.
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| **Modifications**  | The three tiers mentioned above can be used for students with learning disabilities or language barriers as well. These tiers will allow all students an entry point to the lesson. |
| **References** |  Websites for animal videos:Bruce C. Jayne, University of Cincinnati, Videos of Locomotion & Behaviorhttp://www.artsci.uc.edu/collegedepts/biology/fac\_staff/jayne/videos.aspx |
| **Author Info**  | My name is Michael Belcher. I teach Algebra 1 and AP Calculus AB at Hillside New Tech High School in Durham, NC. We are a wall to wall project based learning school. Our goal is to foster academic success through real world applications of the Standard Course of Study. I am currently in my 3rd year of teaching at Hillside New Tech High School and my 6th year of teaching overall. I graduated with a B.A. in Mathematics and minors in Physics and Secondary Education from Wake Forest University in Winston-Salem, NC. I earned an M.A. from Teachers College Columbia University in New York, NY. |

**Using Motion Sensors to Explore Differentiation**

Group Member Name First Role

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**1.** $y=-\frac{1}{2}x+3$ **Graph of the Function**

Script for the Walker:

**Graph of the Velocity (first derivative) Graph of the Acceleration (second derivative)**

**2.** $y=\left|-x+3\right|$ **Graph of the Function**

Script for the Walker:

**Graph of the Velocity (first derivative) Graph of the Acceleration (second derivative)**

**3.** $y=ln\left(x\right)$ **Graph of the Function**

Script for the Walker:

**Graph of the Velocity (first derivative) Graph of the Acceleration (second derivative)**

**4.** $y=-x^{2}+4x$ **Graph of the Function**

Script for the Walker:

**Graph of the Velocity (first derivative) Graph of the Acceleration (second derivative)**

**5.** $f\left(x\right)=\left\{\begin{array}{c}2,\\-x\end{array}\right.\begin{matrix}0\leq x<2\\2\leq x\end{matrix}$ **Graph of the Function**

Script for the Walker:

**Graph of the Velocity (first derivative) Graph of the Acceleration (second derivative)**

**Warm-Up**

Graph each function and find the slope of the tangent line at each of the following points:

1. $x=-2$
2. $x=1$
3. $x=3$
4. $f\left(x\right)=-\left(x-1\right)^{2}$
5. $y=3e^{x-1}$
6. $y=3x\left(x-1\right)^{2}$