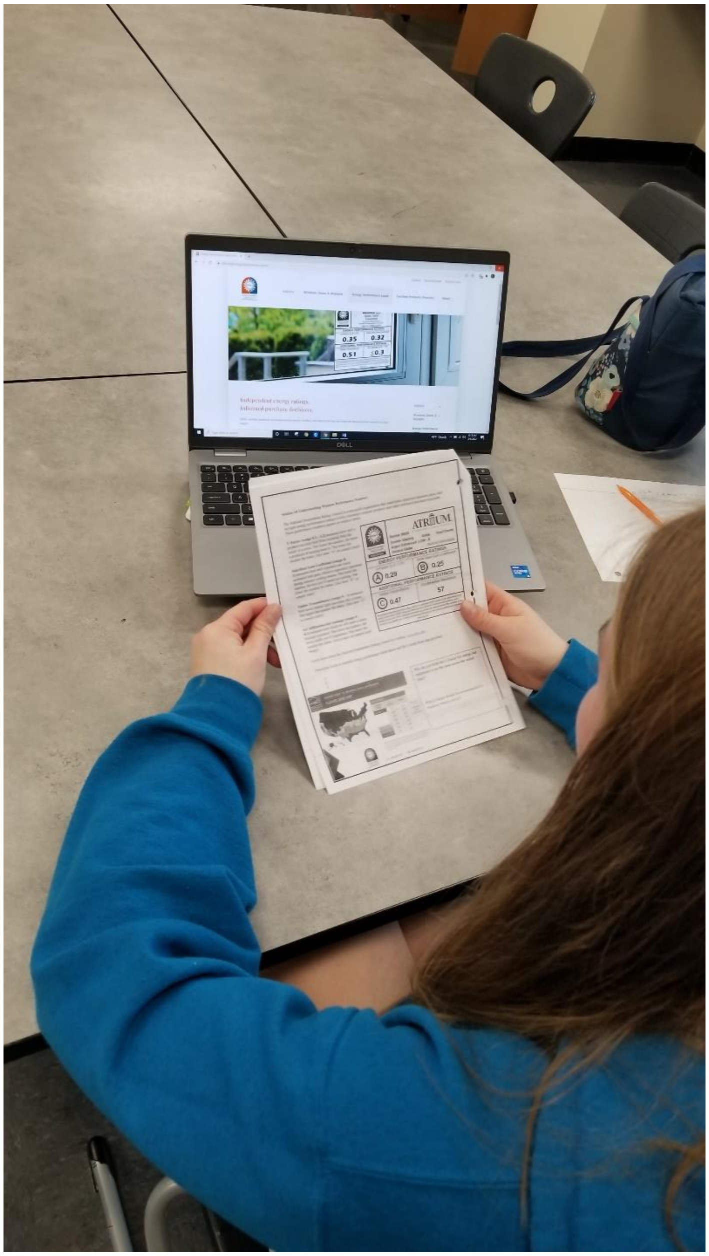
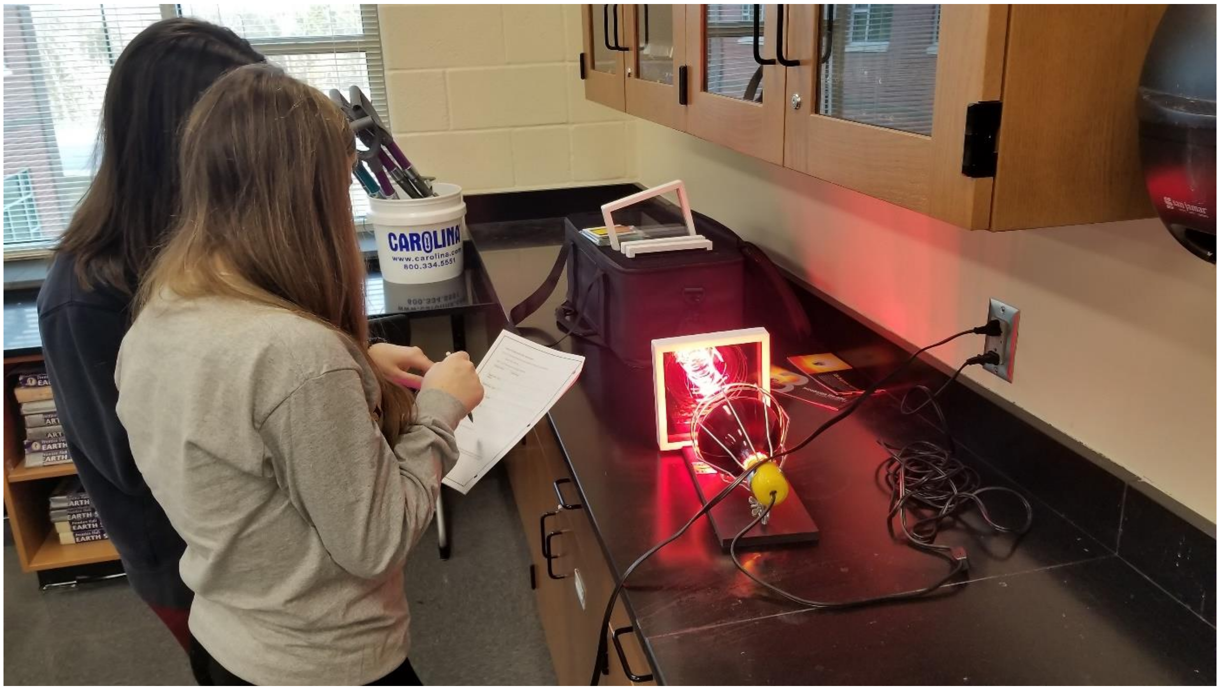
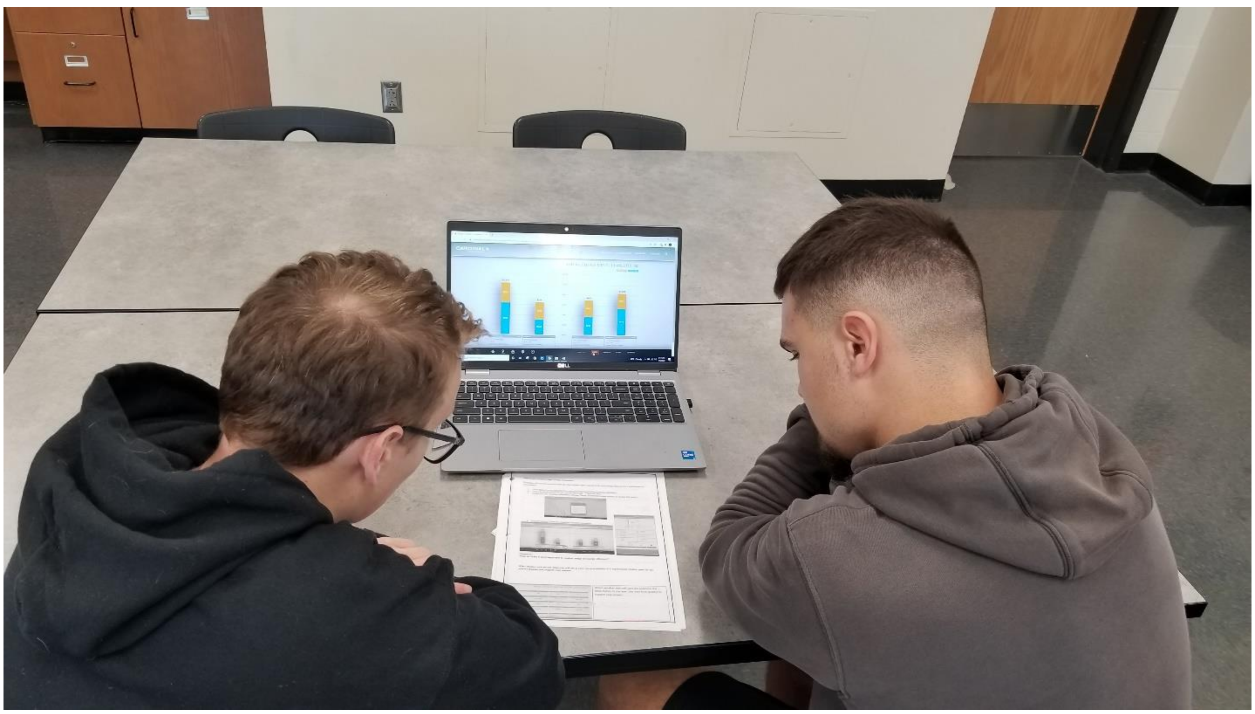
What it looks like in the classroom (Spring 2022 AP Environmental Science).

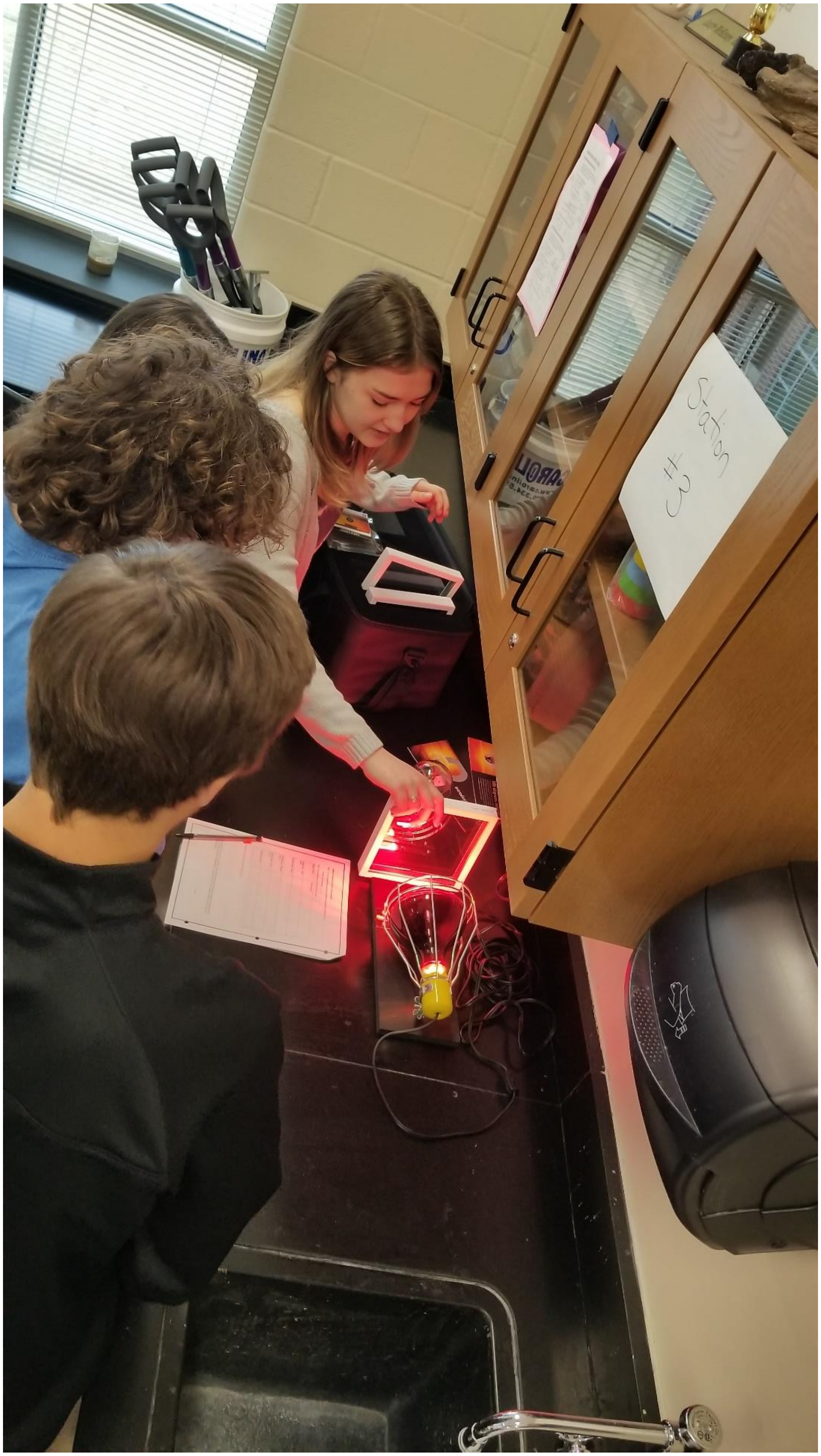



CAROLINA



KENAN FELLOWS PROGRAM
FOR TEACHER LEADERSHIP

NC STATE



Appendix

Know/Need to Know Chart

Student Worksheets

Energy Conversion Answers (National Math & Science Initiative)

REMOVING THE "PANE" FROM HEATING AND COOLING BILLS

ENERGY EFFICIENCY & WINDOW DESIGN

You will be writing a proposal for your principal containing currently installed school window baseline data, energy star efficiency needs for region, and support and reasoning for proposed window replacement.

KNOW	NEED TO KNOW

Energy Conversions & Conservation
Atrium Window & Door

Station #1 Mathematical Conversions

__Conversions__

1 hour = 3600 seconds	1 mile = 5280 feet	1 yard = 3 feet
1 meter = 3.28 feet	1 km = 0.62 miles	1 light second = 300,000,000 meters
1 kg = 2.2 lbs	1 lb = 0.45 kg	1 quart = 0.946 liters
1 m/s = 2.2 miles/hour	1 foot = 12 inches	1 inch = 2.54 cm = 25.4 mm

565,900 seconds into days

17 years into minutes

43 miles into feet

22,647 inches into miles

60 miles per hour into meters per second

130 meters per second into miles per hour

53 yards per hour into inches per week

88 inches per second into miles per day

Station #2 APES ENERGY MATH

Directions: Use the colored packets to walk your way through the APES Math. Remember DO NOT let the math in this class stress you out. You can score a 5 on the AP exam and get ALL the math wrong.

Sample Problem 1

A large, coal-fired electric power plant produces 12 million kilowatt-hours of electricity each day. Assume that an input of 10,000 BTUs of heat is required to produce an output of 1 kWh of electricity.

(a) **Calculate** the number of BTUs of heat needed to generate the electricity produced by the power plant each day.

(b) **Calculate** the number of pounds of coal consumed by the power plant each day, assuming that one pound of coal yields 5,000 BTUs of heat. (Pick up where you left off! Let the problem lead you!)

(c) **Calculate** the number of pounds of sulfur released by the power plant each day, assuming that the coal contains one percent sulfur by weight.

Sample Problem 2

The environmental impact of washing a load of dirty dishes in an electric dishwasher differs from that of washing them in a sink by hand. A comparison of the two methods may allow consumers to spend their money more wisely. Use the assumptions in the table below to perform the calculations that follow.

All the dishes fit into a single load.
The water entering both the water heater and the sink is at 50 °F.
The water heater and the dishwasher are both 100% efficient.
Washing the dishes by hand requires 20 gallons of water heated to 110 °F.
In one complete cycle, the electric dishwasher uses 10 gallons of water heated to 140 °F and the dishwasher also uses 0.500 kilowatt-hour of electrical energy for its mechanical operations.

(a) A British thermal unit or BTU is the amount of energy needed to raise the temperature of one pound of water 1.0 °F. Additionally, one gallon of water has a mass of 8.0 pounds. **Calculate** the total energy used to wash a load of dishes using the dishwasher.

(b) **Calculate** the energy in BTU used to heat the water for washing a load of dishes by hand.

(c) **Calculate** the cost of electricity for each dishwashing method given that one kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is $0.11 per kWh.

Sample Problem 3

Answer the questions below regarding the heating of a house in the Midwestern United States. Use the assumptions in the table below to perform the calculations that follow.

The house has 2,000 square feet of living space.
80,000 BTUs of heat per square foot are required to heat the house for the winter.
Natural gas is available at a cost of $5.00 per thousand cubic feet.
One cubic foot of natural gas supplies 1,000 BTUs of heat energy.
The furnace in the house is 80 percent efficient.

(a) **Calculate** number of cubic feet of natural gas required to heat the house for one winter. Show all the steps of your calculations, including units.

(b) **Calculate** the cost of heating the house for one winter.

VINYL REPLACEMENT WINDOWS

■ Measuring ■ Removal of Old Window ■ Installation Instructions

Station #3 Window Measurement

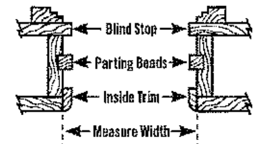
1. **Watch** Atrium Window & Door Measurement 101
2. **Use Measuring Tape to complete measurement on model window.**

BEFORE YOU START! Read these instructions carefully and identify all pieces to be removed or added during installation. All work can be done from inside the house.

HOW TO MEASURE YOUR OPENING TO ORDER THE REPLACEMENT WINDOW

Read ALL of these instructions before measuring. Measure ALL WINDOWS that you intend to replace.

A. CROSS SECTION VIEW OF RIGHT AND LEFT JAMBS



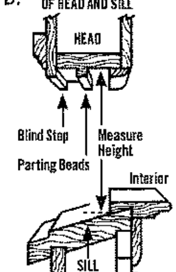
IMPORTANT NOTICE: In ordering windows please provide opening size only. Manufacturing will deduct the following:
WIDTHS: After 1/4" deduction widths will be cut to the nearest 1/8" increment.
HEIGHTS: After 1/4" deduction heights will be cut to the nearest 1/8" increment.

SIZE RESTRICTIONS FOR DOUBLE HUNG: WIDTH: 13" (Minimum) 54" (Maximum); HEIGHT: 19" (Minimum) 91" (Maximum)

WIDTH: Measure between the jambs at 3 points. (Diagram A.) Measure at top, middle and bottom of window opening. Measure from the surface of the jambs, not from the trim strips or parting beads in your old window opening. Use the **SMALLEST** of these three width measurements to order the window!

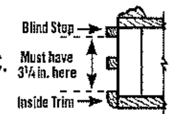
HEIGHT: Measure from the high point of the sill of your window (the surface closest to the inside of the window) to the top of the window opening. (Diagram B.) Make 3 measurements: one at the left side, one at the center, and one at the right side of the window. Use the **SMALLEST** of these three height measurements to order the window! Take these three dimensions for each window you are ordering to your dealer. He will supply you with the correct size windows for your openings.

B. CROSS SECTION VIEW OF HEAD AND SILL



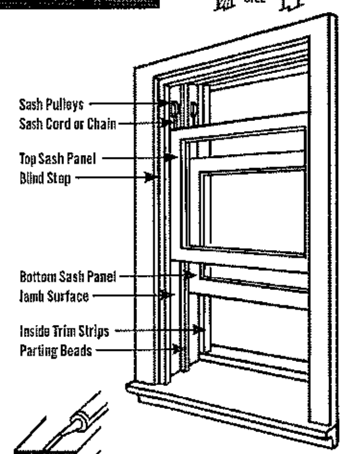
NOTE: Window Rough Opening DEPTH Dimensions Needed (Ignore all pulleys and parting strips when measuring for depth dimensions as they will be removed for replacement window installation).
There must be at least 3 1/4" of opening depth between the inside Window Trim and the outside blind stop strip. (Diagram C.) This much area is needed to accept the depth of the replacement window!

C.



REMOVAL AND INSTALLATION INSTRUCTIONS FOR YOUR NEW SOLID VINYL REPLACEMENT WINDOWS

TOOLS NEEDED: Caulking Gun, Phillips Head Screwdriver, Utility Knife, Hammer, Level, Measuring Tape, Chisel, Electric Drill, Premium Grade Sealant



FROM INSIDE THE HOUSE

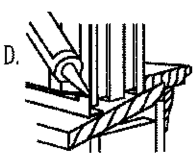
I. REMOVAL OF EXISTING WOOD WINDOW

1. Check new windows before you tear out opening. Make sure you have the correct amounts and sizes.
2. Carefully remove all inside trim or stops (you will reuse this material).
3. Cut bottom sash balance cord or chain and remove sash.
4. Remove the middle parting strips at the sides and top(s).
5. Cut top sash balance cord or chain and remove the sash.
6. Leave exterior stops (blind stops) intact and in place to prevent the replacement window from falling through the opening.

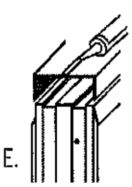
II. PREPARATION OF OPENING AND INSTALLATION PROCESS

1. Clean opening thoroughly and verify integrity of existing wood (replace damaged or rotten components as necessary).
2. Apply a discontinuous bead of sealant to the bottom of the sill angle at two location (this allows water to "weep" away from the window) and place on sill. (Diagram D.)
3. Apply a bead of sealant to the interior of the exterior stop and to the exterior edge of the stool.
4. Apply head expander (if needed) to window using loose fiberglass insulation to fill cavity. (Diagram E.)
5. Set locked window into opening and against wet sealant.
6. Apply a bead of sealant to interior face of window.
7. Replace interior stops (blind stops) against wet sealant.
8. Unlock and raise bottom sash and bottom sash stops. Install screws into pre-drilled installation holes. DO NOT over-tighten screws.
10. Check window frame for square (measure diagonals) and adjust installation screws as necessary.
11. The jamb adjustment grommet screws are located in both side jambs of the window. With a Phillips head screwdriver, turning the screws in a clockwise direction will cause the jamb to "pull" toward the center of the window. This adjustment is necessary to give a closer and even alignment between the sides of the sashes and the jambs, and to ensure proper sash operation. The distance between the operating sashes and the jambs of the window should not exceed 1/16 in.. After making partial adjustment, replace the sashes, operate them and visually judge their alignments with the jambs.
(Adjustment may have to be made several times until satisfactory performance is achieved. You may also make adjustments by loosening and tightening the previously installed jamb screws. In extreme cases the old window opening might be badly "bowed." If the adjustments do not solve this problem, wood shims will have to be used between the rough opening and the replacement window jambs. If the top head expander is used, adjust it up against the rough opening.)
12. Caulk around the entire perimeter of the window on the inside with paintable sealant.

D.



E.



MEASUREMENT NOTES

NO.	STYLE	WIDTH	HEIGHT	LOCATION	QTY.	NO.	STYLE	WIDTH	HEIGHT	LOCATION	QTY.

Station #4 Understanding Window Performance Numbers

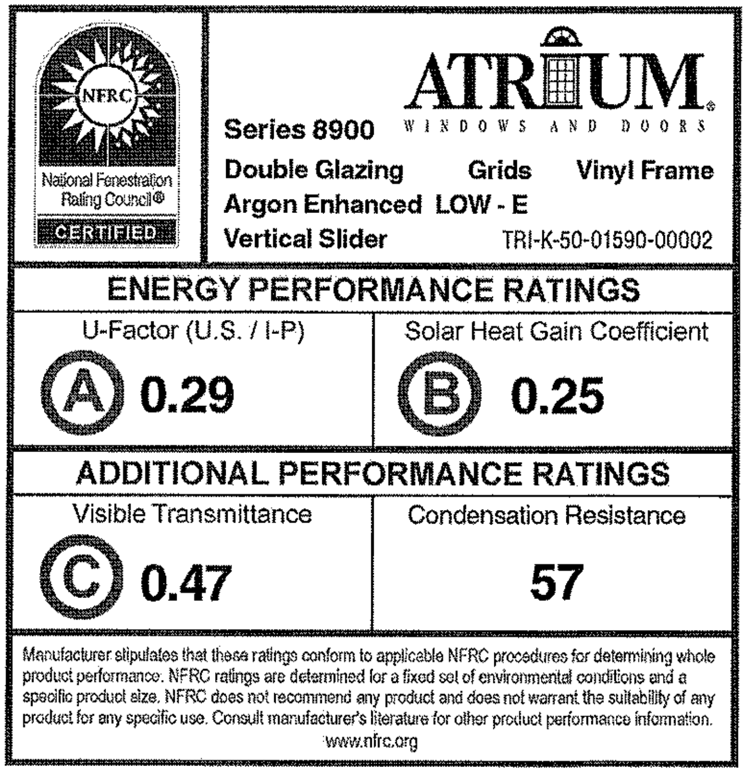
The National Fenestration Rating Council is a non-profit organization that establishes objective window, door, and skylight energy performance ratings to help consumers compare products and make informed purchase decisions. These performance numbers appear on window labels:

U-Factor (range 0.2 – 1.2) measures how well a product can keep heat from escaping from the inside of a room. The lower the number, the better a productis at keeping heat in. The lower the number the better. (*See item "A" on sample label*)

SolarHeat Gain Coefficient (range 0-1) measures how well a product can resist unwanted heat gain, which is especially important during summer cooling season. The lower the number, the less you'll spend on cooling. The lower the number the better. (*See item "B" on sample label*)

Visible Transmittance (range 0 – 1) measures how much natural light can come into a room. The higher the number the better. (*See item "C" on sample label*)

Air Infiltration/Air Leakage (range 0 – 0.3) measures how much air will enter a room through a product. The lower the number, the fewer drafts you'll experience. The lower the number the better. (*Not picture on sample label image*)

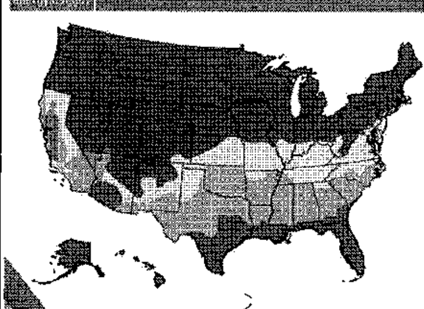


Learn more about the National Fenestration Rating Council by visiting: www.nfrc.org.

Directions: Look at example energy performance sheet above and list 3 trends from data provided:

1.
2.
3.

ENERGY STAR® for Windows, Doors, and Skylights
CLIMATE ZONE MAP



WINDOWS

CLIMATE ZONE	U-FACTOR[1]	SHGC[2]	
Northern	≤0.27	Any	Prescriptive
	=0.28	≥0.32	Equivalent Energy Performance
	=0.29	≥0.37	
	=0.30	≥0.42	
North Central	≤ 0.30	≤ 0.40	
[illegible]	≤ 0.30	≤ 0.25	
Southern	≤ 0.40	≤ 0.25	

NFRC's FenStar® Certification Program ensures windows, doors, and skylights meet ENERGY STAR® specifications and is accredited by the American National Standards Institute (ANSI).

NFRC National Fenestration Rating Council®

301-589-1776 www.nfrc.org

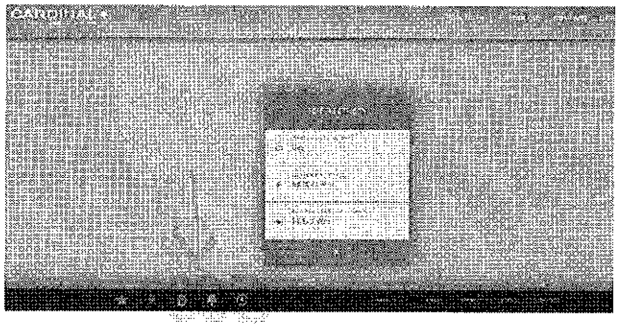
Why do you think the U-Factor for energy star equipment is not the same across the united states?

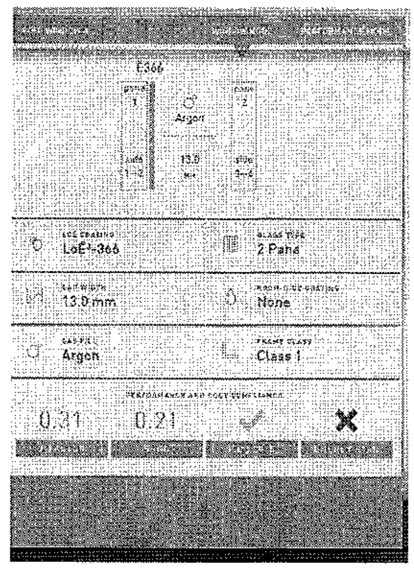
What U-factor should be recommended for windows where you live?

Station #5 Cardinal Glass Energy Calculator

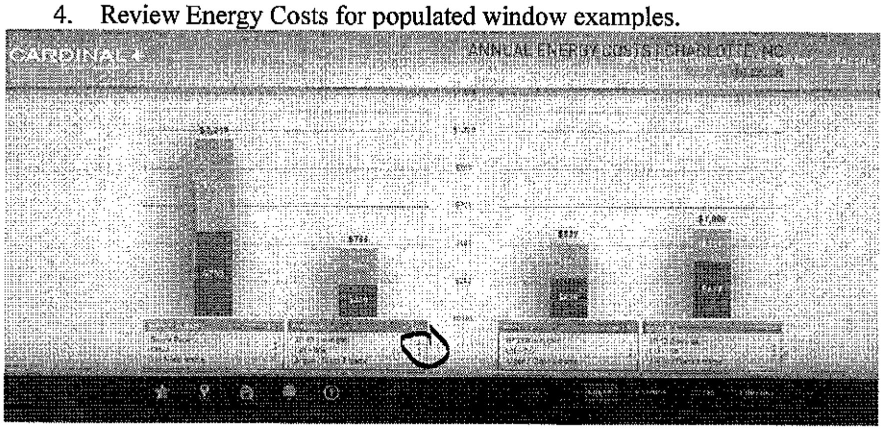
Complete this activity to collect data on what window type is going to be most energy efficient for a replacement at your school.

1. Go to https://www.cardinalcorp.com/technology/applications/energy-calculator/
2. Chose location closest to your school (Example – Charlotte, NC)
3. Customize the building information (House, Shade, Electricity) using buttons on bottom left corner:



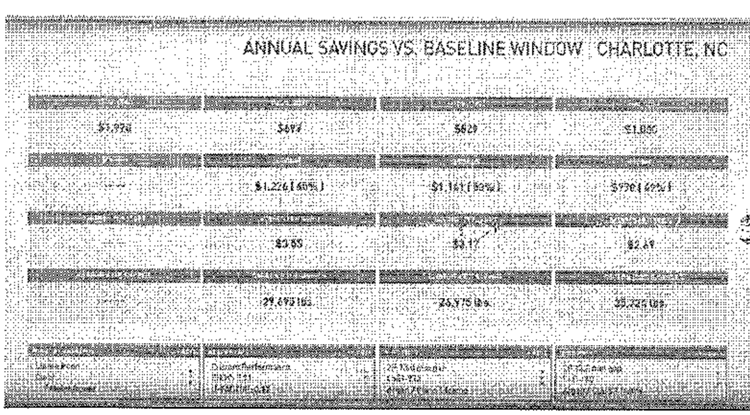


4. Review Energy Costs for populated window examples.



Summary:
What do think is most important in window design for energy efficiency?

What window style do you think you will use in your recommendation of a replacement window pane for the school? Explain and support your answer:



Which window style will save the customer the most money in one year. Use data from graphic to support your answer.

Station #6 Single/Double Pane Measurements

Part 1. Use the single/ double pane tool to collect data on school's current window:

Results: My school has ____

Part 2. Use the Radiometer to test and collect data:

Sample Tested	**Observations**
Single Pane – Your choice	
Double Pane - Clear	
Double Pane – Regular Low- E	
Double Pane – Ultra Low- E	
Triple Pane Ultra	

I predict the most energy efficient window style is:

Atrium Window & Doors also utilizes glass for areas that need specific requirements based on environment like glass to withstand hurricane wind and damage. List and describe one other unique environment that might need specialized glass.

Station #7 Recommendations for Window Replacement

Directions: Write a letter to the principal giving your recommendation for a window replacement plan that will be the most energy efficient while also being cost effective. Select one window in the school that you will collect measurements for and take location and environment into account.

The window we selected is located:____

Space to Collect Data /Measurements	Notes on Design & Layout

Student Resources

- Energy Star for Windows, Doors, and Skylights: Cost and Energy Savings
  - https://www.energystar.gov/ia/partners/manuf_res/windows/ES_Windows_Cost_Energy_Savings.pdf
- Energy Calculator Cardinal Glass Industries
  - https://www.cardinalcorp.com/technology/applications/energy-calculator/

Final Product Space

Recommendation for when windows do need Replaced

NATIONAL MATH + SCIENCE INITIATIVE

AP* Environmental Science

Mastering the Math

Part I: Dimensional Analysis (aka Factor-Label or Unit Cancellation Method)

Sample Problem 1

A large, coal-fired electric power plant produces 12 million kilowatt-hours of electricity each day. Assume that an input of 10,000 BTU's of heat is required to produce an output of 1 kWh of electricity.

(a) **Calculate** the number of BTU's of heat needed to generate the electricity produced by the power plant each day.

(b) **Calculate** the number of pounds of coal consumed by the power plant each day, assuming that one pound of coal yields 5,000 BTU's of heat.

(c) **Calculate** the number of pounds of sulfur released by the power plant each day, assuming that the coal contains one percent sulfur by weight.

Where to start?

Step 1: Ask yourself, "What unit of measure do I need for my answer?"

Read part (a) again. You need to answer in units of "BTU's *per* day". The word "*per*" is code for "divided by" or "in the denominator". Next, translate the words into math. So, "BTUs per day" becomes $\frac{\text{BTU}}{\text{day}}$.

Consider this your *end goal*; it is how you must express your final answer.

Step 2: Am I given any numbers that are paired with the units I seek?

Not exactly! That would be *WAY* too easy. But you can identify a "bridge" of sorts that connects "BTU" and "day". You are given "power plant produces 12 million kilowatt-hours (12,000,000 kWh) of electricity each day" AND "10,000 BTU" of heat is required to produce an output of 1 kWh of electricity". From this we can write two equivalent sets of conversion factors for each of these tidbits of valuable information.

$$\frac{12,000,000\text{ kWh}}{1\text{ day}} \text{ or } \frac{1\text{ day}}{12,000,000\text{ kWh}} \text{ as well as } \frac{1\text{ kWh}}{10,000\text{ BTU}} \text{ or } \frac{10,000\text{ BTU}}{1\text{ kWh}}$$

These are called *conversion factors* and they are your friends! Prepositions are also your friends since they link information together as in "pieces *of* candy" and "at 15 °F". Your objective is to string the conversion factors together (keeping the numbers with the units) so that the *unwanted units* cancel out leaving you with the desired units for your answer. In our example the desired units are $\frac{\text{BTU}}{\text{day}}$. The numerator and denominator of a conversion factor represent the same quantity, such as $\frac{12\text{ inches}}{1\text{ foot}}$ or $\frac{1\text{ foot}}{12\text{ inches}}$ thus we are simply multiplying by "1".

Step 3: Choose the conversion factors AND structure them so that you achieve your "end goal".

Remember we were asked to solve for "BTUs per day" which in "math speak" looks like units of $\frac{\text{BTU}}{\text{day}}$.

It there is a "trick" to this process, it involves *choosing* conversion factors and arranging them so that the *unwanted* units cancel out leaving only the *desired* unit(s) on your final answer! For our example, you want BTU in the numerator of your final answer along with days in the denominator. So, you need to get rid of or cancel the kWh. How?

$\frac{12{,}000{,}000\ \cancel{\text{kWh}}}{1\ \text{day}}$ and $\frac{10{,}000\ \text{BTU}}{1\ \cancel{\text{kWh}}}$ make a nice pair since they result in the desired units of $\frac{\text{BTU}}{\text{day}}$. Yippee!

Step 4: Solve it! Let's revisit our sample problem:

When you have cancelled out the units you don't want and are left only with the units you do want, then you know it's time to multiply all the numerators (top numbers) together, and divide by EACH of the bottom numbers! When setting up a string of conversion factors, it is often helpful to start at the end (units of the desired answer) and work backwards as explained and demonstrated on the screencast.

Sample Problem 1

A large, coal-fired electric power plant produces 12 million kilowatt-hours of electricity each day. Assume that an input of 10,000 BTU's of heat is required to produce an output of 1 kWh of electricity.

(a) **Calculate** the number of BTU's of heat needed to generate the electricity produced by the power plant each day.

$$1\ \cancel{\text{day}} \times \frac{12{,}000{,}000\ \cancel{\text{kWh}}}{1\ \cancel{\text{day}}} \times \frac{10{,}000\ \text{BTU}}{1\ \cancel{\text{kWh}}} = 120{,}000{,}000{,}000\ \text{BTU} = 1.2 \times 10^{11}\ \text{BTU}$$

(b) **Calculate** the number of pounds of coal consumed by the power plant each day, assuming that one pound of coal yields 5,000 BTU's of heat. (Pick up where you left off! Let the problem lead you!)

$$120{,}000{,}000{,}000\ \cancel{\text{BTU}} \times \frac{1\ \text{lb coal}}{5{,}000\ \cancel{\text{BTU}}} = 24{,}000{,}000\ \text{lb coal or } 2.4 \times 10^{7}\ \text{lbs of coal}$$

(c) **Calculate** the number of pounds of sulfur released by the power plant each day, assuming that the coal contains one percent sulfur by weight.

$$\frac{24{,}000{,}000\ \cancel{\text{lb coal}}}{\text{day}} \times \frac{1\ \text{lb Sulfur}}{100\ \cancel{\text{lb coal}}} = \frac{240{,}000\ \text{lbs of sulfur}}{\text{day}} \text{ or } \frac{2.4 \times 10^{5}\ \text{lbs of sulfur}}{\text{day}}$$

OR

Realize that 1.0% is equal to 0.01 in decimal form and simply multiply

$$\therefore 0.01 \times \frac{24{,}000{,}000\ \text{lb coal}}{\text{day}} \text{ becomes equal to } \frac{240{,}000\ \text{lbs of sulfur}}{\text{day}} \text{ or } \frac{2.4 \times 10^{5}\ \text{lbs of sulfur}}{\text{day}}$$

Sample Problem 2

The environmental impact of washing a load of dirty dishes in an electric dishwasher differs from that of washing them in a sink by hand. A comparison of the two methods may allow consumers to spend their money more wisely. Use the assumptions in the table below to perform the calculations that follow.

All the dishes fit into a single load.
The water entering both the water heater and the sink is at 50 °F.
The water heater and the dishwasher are both 100% efficient.
Washing the dishes by hand requires 20 gallons of water heated to 110 °F.
In one complete cycle, the electric dishwasher uses 10 gallons of water heated to 140 °F and the dishwasher also uses 0.500 kilowatt-hour of electrical energy for its mechanical operations.

(a) A British thermal unit or BTU is the amount of energy needed to raise the temperature of one pound of water 1.0 °F. Additionally, one gallon of water has a mass of 8.0 pounds. **Calculate** the total energy used to wash a load of dishes using the dishwasher.

(b) One kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is $0.11 per kWh. **Calculate** the energy in BTU used to heat the water for washing a load of dishes by hand.

(c) **Calculate** the cost of electricity for each dishwashing method given that one kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is $0.11 per kWh.

Step 1: Ask yourself, "What unit of measure do I need for my answer?"

Read part (a) again. The question asks you to calculate "total energy" per load, so you need to answer in units of "BTU per load". Translate that to math and you get $\frac{\text{BTU}}{\text{load}}$ or simply BTU. Consider this your *end goal*; it is how you must express your final answer for parts (a). Next, read part (b) again. The final answer to part (b) should also be reported in $\frac{\text{BTU}}{\text{load}}$ or simply BTU.

Step 2: Am I given any numbers that are paired with the units I seek?

Yes! (Remember, it is a legal maneuver to "flip" any of these conversion factors as long as you keep the appropriate number with the appropriate unit!)

$$\frac{10\text{ gal } H_2O}{\text{load}} \text{ and } \frac{1\text{ gal } H_2O}{8\text{ lb } H_2O} \text{ and } \frac{1\text{ BTU}}{1\text{ lb } H_2O \bullet 1\text{ °F}}$$ and *note the temperature change*!

Yep! That's TWO items in the denominator of one of the conversion factors, thus twice the cancelling fun.

Step 3: Choose the conversion factors AND structure them so that you achieve your "end goal".
Ask yourself "What to flip and why flip at all?"

Step 4: Solve it!

Sample Problem 2

The environmental impact of washing a load of dirty dishes in an electric dishwasher differs from that of washing them in a sink by hand. A comparison of the two methods may allow consumers to spend their money more wisely. Use the assumptions in the table below to perform the calculations that follow.

All the dishes fit into a single load.
The water entering both the water heater and the sink is at 50 °F.
The water heater and the dishwasher are both 100% efficient.
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In one complete cycle, the electric dishwasher uses 10 gallons of water heated to 140 °F and the dishwasher also uses 0.500 kilowatt-hour of electrical energy for its mechanical operations.

(a) A British thermal unit or BTU is the amount of energy needed to raise the temperature of one pound of water 1.0 °F. Additionally, one gallon of water has a mass of 8.0 pounds. **Calculate** the total energy used to wash a load of dishes using the dishwasher.

Start with the temperature change! Why? It has an understood denominator of "1" which doesn't need cancelling out.

$$90\ \cancel{°F} \times \frac{1\ \text{BTU}}{1\ \cancel{\text{lb}\ H_2O} \bullet 1\ \cancel{°F}} \times \frac{10\ \cancel{\text{gal}\ H_2O}}{\text{load}} \times \frac{8\ \cancel{\text{lb}\ H_2O}}{1\ \cancel{\text{gal}\ H_2O}} = 7,200\ \frac{\text{BTU}}{\text{load}}$$

(b) **Calculate** the energy in BTU used to heat the water for washing a load of dishes by hand.

$$60\ \cancel{°F} \times \frac{1\ \text{BTU}}{1\ \cancel{\text{lb}\ H_2O} \bullet 1\ \cancel{°F}} \times \frac{20\ \cancel{\text{gal}\ H_2O}}{\text{load}} \times \frac{8\ \cancel{\text{lb}\ H_2O}}{1\ \cancel{\text{gal}\ H_2O}} = 9,600\ \frac{\text{BTU}}{\text{load}}$$

(c) **Calculate** the cost of electricity for each dishwashing method given that one kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is $0.11 per kWh.

Dishwasher: $7,200\ \frac{\cancel{\text{BTU}}}{\text{load}} \times \frac{1\ \cancel{\text{kWh}}}{3,400\ \cancel{\text{BTU}}} = \frac{2.12\ \text{kWh}}{\text{load}} + \frac{0.5\ \text{kWh}}{\text{load}} = \frac{2.62\ \cancel{\text{kWh}}}{\text{load}} \times \frac{\$0.11}{1\ \cancel{\text{kWh}}} = \frac{\$0.29}{\text{load}}$

Sink: $9,600\ \frac{\cancel{\text{BTU}}}{\text{load}} \times \frac{1\ \cancel{\text{kWh}}}{3,400\ \cancel{\text{BTU}}} \times \frac{\$0.11}{\cancel{\text{kWh}}} = \$0.31$ or 31 cents

Surprising? Remember, these problems ignore the price of the water!

Part 2: Determining Energy Efficiency

Energy efficiency is simply the ratio of work output to work input. *If* the problem states that system is less than 100 % efficient, then the energy output must be *divided by* the stated efficiency to determine how much energy input is needed.

$$\text{Percent Efficiency} = \frac{\text{Work}_{\text{out}}}{\text{Work}_{\text{in}}} \times 100\%$$

Our previous sample problem made this easy on us by telling us the efficiency is 100%. Don't always expect that since such machines do not exist! If a machine or process is 75% efficient then you simply multiply anything you calculate as being "perfect" by 0.75 effectively reducing it to only 75% rather than leaving it at the elusive 100% efficiency.

Sample Problem 3

Answer the questions below regarding the heating of a house in the Midwestern United States. Use the assumptions in the table below to perform the calculations that follow.

The house has 2,000 square feet of living space.
80,000 BTUs of heat per square foot are required to heat the house for the winter.
Natural gas is available at a cost of $5.00 per thousand cubic feet.
One cubic foot of natural gas supplies 1,000 BTUs of heat energy.
The furnace in the house is 80 percent efficient.

(a) **Calculate** number of cubic feet of natural gas required to heat the house for one winter. Show all the steps of your calculations, including units.

$$2.0\times10^3\ \cancel{ft^2}\times\frac{8.0\times10^4\ \cancel{BTU}}{\cancel{ft^2}}\times\frac{1\,ft^3}{1\times10^3\ \cancel{BTU}}=\frac{16\times10^7}{1\times10^3}ft^3=\frac{1.6\times10^8}{1\times10^3}ft^3=1.6\times10^{(8-3)}ft^3=1.6\times10^5\,ft^3$$

But wait! That's only if the process is 100% efficient. Since the process is only 80% efficient it will require *more* natural gas to heat the house. So, divide your "perfect" answer by the efficiency expressed as a decimal value, 0.80 in our case.

$$1.6\times10^5\,ft^3\div0.80=\frac{1.6}{0.8}\times10^5\,ft^3=2\times10^5\,ft^3 \text{ or } 200{,}000\ ft^3 \text{ if you prefer}$$

(b) **Calculate** the cost of heating the house for one winter.

$$2\times10^5\ \cancel{ft^3}\times\frac{\$5.00}{10^3\ \cancel{ft^3}}=\$10\times10^{(5-3)}=\$10\times10^2=\$1{,}000$$

Part 3: Scientific Notation

Scientific notation is used to communicate extremely large numbers such as the speed of light or extremely small numbers such as the radius of a blood cell.

1. ***Converting decimal numbers LESS THAN ONE to Scientific Notation***

Example: Express the number 0.00234 in scientific notation.

- Place a decimal after the first nonzero number. We refer to this as the *coefficient*. *Coefficients* need to be between 1 and 9. The *coefficient* is then multiplied by ten raised to an *exponent*, $10^{exponent}$
- Determine the *exponent* on the "10" by counting the number of places you move the decimal point. If you move the decimal to the right, the *exponent* will be negative. If you move the decimal point to the left, the *exponent* will be positive.

  Negative exponents tell us how many times the number has been **divided by 10**
  Positive exponents tell us how many times the number has been **multiplied by 10**

So, to express the number 0.00234 in scientific notation, we move the decimal 3 places to the right: 0.00234 to give 2.34×10^{-3}

What does this *really* mean mathematically? $\frac{2.34}{10 \times 10 \times 10}$ or $\frac{2.34}{10^3}$ or $\frac{2.34}{1{,}000}$ or 2.34×10^{-3}

Essentially, it means the number is small. How small? In the thousandths.

Why do this? For years you've thought 0.00234 was a perfectly good number just the way it is. Why convert it to scientific notation? BECAUSE it allows us to focus on the "order of magnitude" of *measurements* or *quantities* rather than the coefficients.

Compare 2.34×10^{-3} to 2.34×10^{3}

It's not that the 2.34 is the most important part of the number—it's that the number is in the **thousandths** (10^{-3}) versus the **thousands** (10^3). So, 2.34×10^3 is SIX power of ten's ($10 \times 10 \times 10 \times 10 \times 10 \times 10$), that's 10^6, or a million times larger! We say that 2.34×10^3 is six orders of magnitude greater than 2.34×10^{-3}

2. ***Nuances of Adding and Subtracting Numbers Written in Scientific Notation***

First, make sure both numbers have the same exponent on the "× 10" part. It's easiest to convert one number to the greater exponent:

$5.2 \times 10^3 + 3.6 \times 10^4$ becomes $0.52 \times 10^4 + 3.6 \times 10^4$ or, you may prefer it stacked

$$\begin{array}{r} 0.52 \times 10^4 \\ +\ 3.6\ \ \times 10^4 \\ \hline 4.12 \times 10^4 \end{array}$$

Either way, $(0.52 + 3.6) \times 10^4$ becomes 4.12×10^4

All done! Subtraction works the same way.

3. ***Nuances of Multiplying Numbers Written in Scientific Notation:***

This is much easier! Simply, *multiply the coefficients* and then *add the exponents* on the power of 10.

$(4 \times 10^{-2}) \times (2 \times 10^{10}) = (4 \times 2) \times 10^{(-2+10)}$ becomes $8 \times 10^{(-2+10)}$ which simplifies to 8×10^{8}

Division works the same way EXCEPT you *subtract the exponents* on the power of 10.

$(4 \times 10^{-2}) \div (2 \times 10^{10}) = (4 \div 2) \times 10^{(-2-10)}$ becomes $2 \times 10^{(-2-10)}$ which simplifies to 2×10^{-12}

Sample Problem 4

Calculate the potential reduction in petroleum consumption in gallons of gasoline per year that could be achieved in the United States by introducing electric vehicles under the following assumptions:

- The mileage rate for the average car is 25 miles per gallon of gasoline.
- The average car is driven 10,000 miles per year.
- The United States has 150 million cars.
- Ten percent of the US cars could be replaced with electric cars.

Start with the easy simplification & translate the numbers into scientific notation:

10% of 150 million cars = 15 million cars = 1.5×10^{7} cars… now on to the dimensional analysis…

$$1.5\times10^{7}\ \cancel{\text{cars}} \times \frac{1\times10^{4}\ \cancel{\text{mi}}}{\cancel{\text{car}} \bullet \text{year}} \times \frac{1\text{ gal}}{25\ \cancel{\text{mi}}} = \frac{1.5\times10^{11}}{25}\frac{\text{gal}}{\text{year}} \text{ then simplify to } \frac{150}{25}\times10^{9}\frac{\text{gal}}{\text{year}} = 6\times10^{9}\frac{\text{gal}}{\text{year}}$$

Part 4: Population Growth Rate

Demography is the study of the characteristics of human populations, such as size, growth, density, distribution, and other vital statistics. In demography, population growth is used informally for the more specific term *population growth rate* (*PGR*), and is often used to refer specifically to the growth of the human population of the world.

The most common way to express population growth is as a percentage, *not* as a rate. First, we calculate the *change* in a population using the difference between the crude birth and death rates. What are those? The *crude birth rate* (*CBR*) is the total number of births per year per 1,000 people and the crude death rate (*CDR*) is the total number of deaths per year per 1,000 people. In the formula below we simply divide by ten to report the *PGR* as a percentage. (We wouldn't need to divide by 10 if the CBR & CDR were expressed per "100" instead of per 1,000.)

$$PGR \text{ (as a percentage)} = \frac{CBR - CDR}{10}$$

Part 5: The Rule of 70

The Rule of 70 is used to calculate the *time* required for a *doubling of a population* based upon population growth rate, *PGR,* expressed as a percentage. *Resist the urge to express the growth percentage as a decimal value, leave it as a percentage!*

The Rule of 70 is mathematically expressed as:

$$time_{doubling} = \frac{70}{PGR \text{ (expressed as a percentage)}}$$

How long will it take for a city with an annual population growth rate of 5% to double its population?

$$time_{doubling} = \frac{70}{PGR \text{ (expressed as a percentage)}} = \frac{70}{5\%} = 14 \text{ years}$$

How long will it take for the same city if the growth rate were 7%?

$$time_{doubling} = \frac{70}{PGR \text{ (expressed as a percentage)}} = \frac{70}{7\%} = 10 \text{ years}$$

Sample Problem 5

The fictional country **Industria** is tracking its population data. In 1855, the first year vital statistics were reported for the country, the population was 1.6 million, with a crude birth rate of 43 per 1,000. At that time the population of Industria was growing quite slowly, because of the high death rate of 41 per 1,000. In 1875 the population began to grow very rapidly as the birth rate remained at the 1855 level. While the crude death rate dropped dramatically to 20 per 1,000. Population growth continued to increase in the small country into the late 1800's, even though birth rates began to decline slowly.

In 1895 the crude birth rate had dropped to 37, and the crude death rate to 12 per 1,000. A complete census was also conducted and revealed that the population of Industria had grown to 2.5 million. By 1950 population growth gradually began to decline as the death rate remained at its 1895 level, while the birth rate continued to decline to 22 per 1,000. In 1977 vital statistics revealed that the death rate was 10 per 1,000, and that population growth had slowed even more to an annual rate of 0.4%. By 1990 Industria had reduced its birth rate to that of its now constant, low death rate, and the population transition was complete.

(a) **Calculate** the annual growth rate of Industria in 1950.

$$\text{Annual } PGR \text{ (as a percentage)} = \frac{CBR - CDR}{10} = \frac{22-12}{10} = 1\%$$

(b) **Calculate** the birth rate in Industria in 1977.

In 1977 CDR was 10 and the PGR was 0.4% . So, solve for *CBR*:

$$PGR = \frac{CBR - CDR}{10} \therefore 0.4\% = \frac{CBR - 10}{10} \therefore 4 = CBR - 10 \therefore CBR = 14$$

continued on next page

(c) Assume the population of Industria continues to maintain the population growth rate recorded for the country in 1895. **Determine** the year in which the expected size of the population will have quadrupled.

First, calculate the PGR for 1895: $PGR_{1895} = \frac{CBR - CDR}{10} = \frac{37-12}{10} = \frac{25}{10} = 2.5\%$

Next, use the Rule of 70 to determine the number of years it takes the population to double:

$$time_{doubling} = \frac{70}{PGR \text{ (expressed as a percentage)}} = \frac{70}{2.5\%} = 28 \text{ years}$$

Finally, realize that a quadrupling is 2 doublings, so 56 years have passed by the time the size of the population has doubled. So, 1895 + 56 additional years = 1951
(Harry S. Truman was president, I Love Lucy made its TV debut, and the Korean War was taking place.)

Part 6: Per Capita

Per Capita is a Latin term that translates into "by head," basically meaning "average per person." Per capita can even the place of saying "per person" in any number of statistical observances. In most cases the term is used in relation to economic data or reporting, but can also be used in most any other occurrence of population description.

Sample Problem 6
Between 1950 and 2000, global meat production increased from 52 billion kilograms to 240 billion kilograms. During this period, the global human population increased from 2.6 billion to 6.0 billion.

(a) **Calculate** the per capita meat production in 1950 and in 2000.

For 1950: $\frac{52\times10^9\text{kg}}{2.6\times10^9} = \frac{52\times10^9\text{kg}}{26\times10^8} = 2\times10^{(9-8)}\text{kg} = 2\times10^1\text{kg} = 20 \text{ kg per capita}$

For 2000: $\frac{240 \times 10^9 \text{ kg}}{6 \times 10^9} = 40 \text{ kg per capita}$

(b) **Calculate** the change in global per capita meat production during this 50-year period. Express your answer as a percentage.

No calculation necessary! Since the consumption per capita doubled, that translates to a 100% increase.

Sample Problem 7

West Fremont is a community consisting of 3,000 homes. A small coal-burning power plant currently supplies electricity for the town. The capacity of the power plant is 12 megawatts (MW) and the average household consumes 8,000 kilowatt hours (kWh) of electrical energy each year. The price paid to the electric utility by West Fremont residents for the energy is $0.10 per kWh. The town leaders are considering a plan, the West Fremont Wind Project (WFWP), to generate their own electricity using 10 wind turbines that would be located on the wooded ridges surrounding the town. Each wind turbine would have a capacity of 1.2 MW and each would cost the town $3 million to purchase, finance, and operate for 25 years.

(a) Assuming that the existing power plant can operate at full capacity for 8,000 hrs/yr, how many kWh of electricity can be produced by the plant in a year?

$$\frac{8{,}000\text{ hr}}{\text{yr}} \times 12\times10^3\text{ kW} = 9.6\times10^7\ \frac{\text{kWh}}{\text{yr}}$$

(b) At the current rate of electricity energy per household, how many kWh of energy does the community consume in one year?

$$3{,}000\text{ homes}\times\frac{8{,}000\text{ kWh}}{\text{yr}} = 24{,}000{,}000\frac{\text{kWh}}{\text{yr}}\text{ or } 2.4\times10^7\frac{\text{kWh}}{\text{yr}}$$

(c) Assuming that the electrical energy needs of the community do not change during the 25-year lifetime of the wind turbines, what would be the cost to the community of the electricity supplied by the WFWP over 25 years? Express your answer in dollars/kWh.

TWO DIFFERENT METHODS earn full credit: (surprising since the costs are very different!)

$$2.4\times10^7\frac{\text{kWh}}{\cancel{\text{yr}}}\times 25\ \cancel{\text{yrs}} = 6.0\times10^8\text{ kWh and } 10\ \cancel{\text{turbines}}\times\frac{\$3\times10^6}{\cancel{\text{turbines}}} = \$3\times10^7$$

$$\therefore\ \text{Cost is } \frac{\$3\times10^7}{6.0\times10^8\text{ kWh}} = \frac{\$3\cancel{\times10^7}}{60\times\cancel{\times10^7}\text{ kWh}} = \frac{\$3}{60} = \frac{\$0.05}{\text{kWh}}$$

OR

$$9.6\times10^7\frac{\text{kWh}}{\cancel{\text{yr}}}\times 25\ \cancel{\text{yrs}} = 2.4\times10^9\text{ kWh}$$

$$\therefore\ \text{Cost is } \frac{\$3\times10^7}{2.4\times10^9\text{ kWh}} = \frac{\$3\times10^7}{2.4\times10^9\text{ kWh}} = \frac{\$3}{240} = \frac{\$0.0125}{\text{kWh}}$$

Sample Problem 8

The Cobb family of Fremont is looking at ways to decrease their home water and energy usage. Their current electric hot-water heater raises the water temperature to 140° F, which requires 0.20 kWh/gallon at a cost of \$0.10/kWh. Each person in the family of four showers once a day for an average of 10 minutes per shower. The shower has a flow rate of 5.0 gallons per minute.

(a) Calculate the total amount of water that the family uses per year for taking showers.

$$4\ \cancel{\text{people}} \times 365\ \frac{\cancel{\text{days}}}{\text{yr}} \times \frac{1\ \cancel{\text{shower}}}{\cancel{\text{person}} \bullet \cancel{\text{day}}} \times \frac{10\ \cancel{\text{min}}}{\cancel{\text{shower}}} \times \frac{5\ \text{gal}}{\cancel{\text{min}}} = 73{,}000\ \frac{\text{gal}}{\text{yr}}$$

(b) Calculate the annual cost of the electricity for the family showers, assuming that 2.5 gallons per minute of the water used is from the hot-water heater.

So, they are telling us that ½ of the *total* water used in each shower is water heated by the hot water heater. Fair enough! Simply start this cost analysis using ½ (73,000) = 36,500 $\frac{\text{gal}}{\text{yr}}$. So,

$$36{,}500 \frac{\cancel{\text{gal}}}{\text{yr}} \times \frac{0.20\ \cancel{\text{kWh}}}{\cancel{\text{gal}}} \times \frac{\$0.10}{\cancel{\text{kWh}}} = \frac{\$36{,}500 \times (0.20 \times 0.10)}{\text{yr}} = \frac{\$730}{\text{yr}}$$

(c) The family is considering replacing their current hot-water heater with a new energy-efficient hot-water heater that costs \$1,000 and uses half the energy that their current hot-water heater uses. How many days would it take for the new hot-water heater to recover the \$1,000 initial cost?

$$\text{Cost of OLD: } \frac{\$730}{365\ \text{days}} = \frac{\$2}{\text{day}} \text{ versus Cost of NEW: Half as much } \therefore \frac{\$1}{\text{day}}$$

So, the family is saving a \$1 a day, thus it will take 1,000 days to save \$1,000 which recoups their original investment!