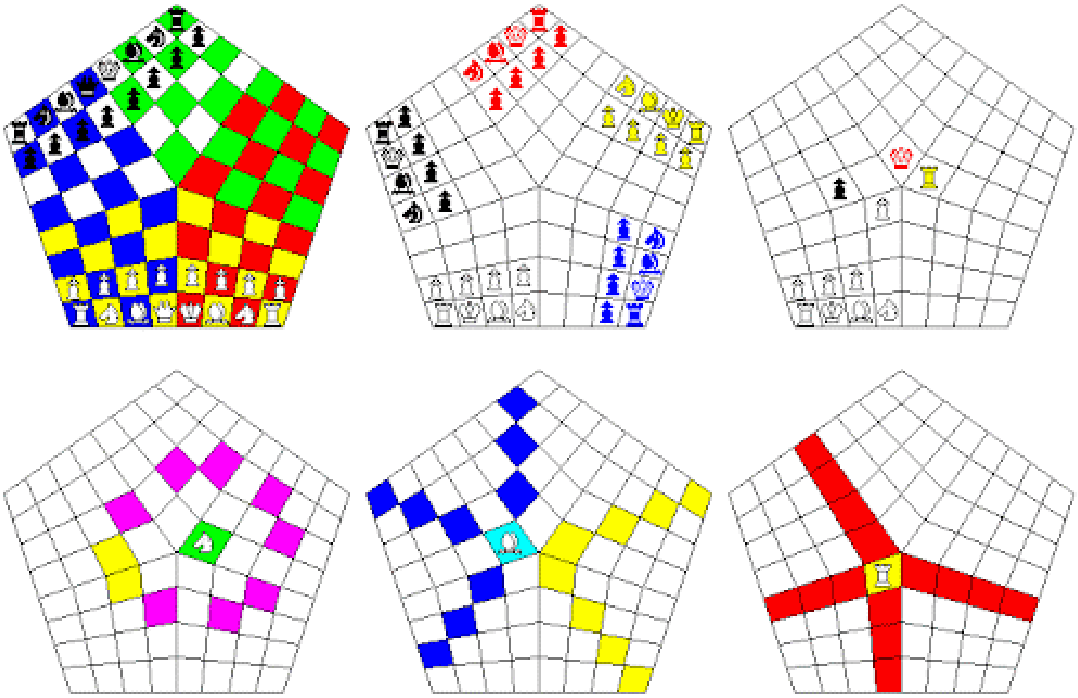
KENAN FELLOWS PROGRAM
FOR TEACHER LEADERSHIP

NC STATE



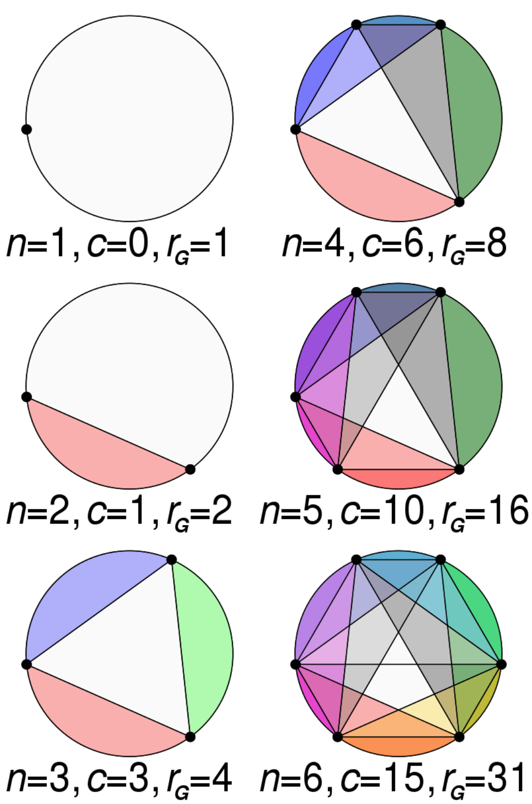
Computational Thinking & Problem Solving in the Secondary Context

OVERVIEW

In this lesson, students will develop their problem solving skills by thinking computationally. Students are presented with challenges that require students to develop generalized strategies for addressing novel problems beyond the immediate scope of their understanding. Through the development of computational thinking, students are encouraged to step outside of their intellectual comfort zones and approach cognitive adversity with resilience.

AUTHOR	GRADE LEVEL	CONTENT AREA
Rick Lage	9-12	Secondary Science

ESSENTIAL QUESTIONS	TIME NEEDED	STANDARDS
How should I approach a problem I don't already know how to solve? How can things I do know help me solve new problems? What does it mean to generalize a problem solving process? How can problem solving algorithms help me better understand the way a problem works?	Prep: 30-45 min. Learn: ~2 90 minute periods (can be modified for a single class or 3 day learning unit) Wrap Up: 15-20 minutes	I incorporated this lesson into my scientific practices unit at the beginning of chemistry and physics classes. However, I think this could be adapted to a wide range of standards by adjusting problem types to suit the learning unit.



Making **Connections**

Prior to this lesson, students have been refreshed on some of the fundamentals of scientific inquiry. They also will be aware of nearly all the mathematical skills required to approach the problems presented to them such as solving 2-step equations, areas of simple geometric figures, basic probability, and others. Students will be asked to utilize those skills in conjunction with computational thinking and generalization outside of the scope of normal STEM instruction.

Background

This lesson opens with the instructor facilitating a conversation about how to solve a problem for which students don't already know the answer. This includes conversations about real-world problems and skills needed to adapt when there is not a predefined algorithm in place. Students are strategically placed into small groups. The size and strategy will likely vary, depending on the instructor's particular needs – I worked with strategically well-rounded trios. Students are then asked to secure their phones and chromebooks to avoid internet access as a solution to their forthcoming problems. Then, students are given their problem-solving challenge and resources to help them work those problems. It is important that students are reminded that they can work the problems in any order and that evidence of their process is the primary objective for submission of the assignment (over final answers). For my group, this assignment was conducted with a problem set sufficient to allow for two days of exploration, so at the end of day one, I collected students' problems and work thus far. On the second day, problems were returned and before the period ended, we came together for a debrief on problem-solving processes and the strategies students found insightful.

Materials

For this assignment I utilized:

- Printed handouts
- Chart Paper
- Rulers
- Calculators

If it would be advantageous for your instruction, it would be easy to substitute:

- Technology boards (e.g. SMART, Brightlink)
- Desmos
- Geogebra
- White Boards
- Virtual Collaboration tools (Padlet, Nearpod, etc.)

The **Activity**

Part 1: Opening (10-15 min)

- Welcome students to class & the independent warm up prompt "How do you attempt to solve a problem you've never seen before?"
- Allow students time to think and compose short replies
- Guide a class discussion on the prompt, allowing students to share and productively reply to each other's answers.

Part 2: The Challenge

- Introduce the problem-solving challenge, being sure to give an overview of the rules, whether the challenge is collaborative or competitive, and what resources are and are *not* allowed while students attempt to solve their problems.
- Be sure to emphasize the problem-solving process as more important than any one solution.
  - I utilized a phrasing something like: "Most of the problems are quite advanced and I wouldn't be at all disappointed if you didn't get a single one correct. The point of this exercise is to try your best and think about how you solve problems you don't already know how to approach."
- Overview of grouping strategy:
  - As mentioned above, I think the educator will have the best insight into what is most productive & appropriate for their group of students. However, I do think this activity works best when students have at least one partner.
- Distribute the problem sets and provide access to the tools students may use to tackle their problems.
  - I believe that providing students with small hints or validation of their problem-solving process can be very helpful in preventing students from getting overly discouraged.

Again, depending on your group, you may want to give each group a set number of ‘hints’ to cash in or simply provide minor feedback as students encounter obstacles and/or get discouraged.

- As mentioned above, I split this across two days so I could collect each group's work into folders between class periods to discourage hunting for answers online.

Part 3: Debrief

- As a closure, bring students back together to discuss the problem strategies they found productive and unproductive.
- Remember to emphasize the algorithmic process of generalizing a problem-solving case.
- It may also be helpful to be ready to provide solutions to any of the puzzles students were stuck on, as usually there are a few that students are dying to know the answer to.
- It may also be helpful to close a class period by collecting students' work and opening the next period with this wrap up. This way, you can collect and provide feedback on their work and reference that during the wrap-up discussion.

WRAP UP **AND** ACTION

Students were assessed both formally and informally in this lesson. In my case, students received a formal grade based on their effort during the chalance. Students with the most points earned solving puzzles were able to earn a few additional extra points (which helped motivate groups to solve the problems to the best of their ability) on other assignments. Additionally, students are informally assessed through their communication, collaboration and problem solving skills by the instructor as they work on their challenges.

Extensions

Additional opportunities include the school-wide puzzle challenges and other puzzle, codebreaking, and problem solving challenges for students such as the Mathworks Math Modeling Challenge, the North Carolina State Math Competition, and others.

Resources

In working on these resources, I learned a lot from the published works of Martin Gardner, renowned puzzle author, mathematician, and Scientific American columnist.

The YouTube channels Numberphile and Computerphile both also proved valuable in developing many of these materials.

Code.org, codecademy, khanacademy, and stackoverflow all were instrumental in coming to understand some of the basics of computer programming and generalizing a problem to be solved by an algorithm.

About the Author

Rick Lage is a science teacher, NCDPI North East Computer Science Kenan Fellow, whose internship took place at TCOM in Elizabeth City North Carolina & who finds third person autobiographical writing deeply uncomfortable.

About the Fellowship

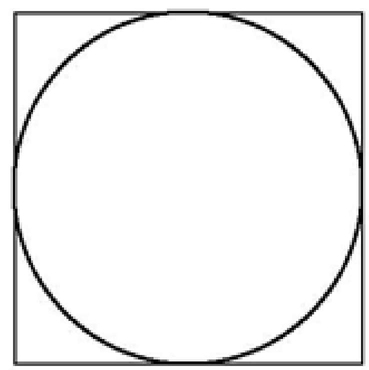
My internship took place at the aerospace technology company, TCOM, where my mentor was Rick Evens. During my internship, I was able to learn about the operations and organization of the international engineering, programming, and manufacturing effort that comes along with TCOM aerostat systems. While many of the particulars of this process are privileged information, I was able to focus on how problem-solving and computational skills were implemented and in-demand across TCOM's many operating departments and levels. While these interviews yielded many take-aways, one thing that came up throughout nearly every part of my internship was the need for adaptive thinking and the ability to generalize problem solving. Repeatedly, across multiple fellowship and internship environments, it was echoed that ‘we can teach them the particulars’ but that it was far more important to future engineers and programmers to be able to adapt and generalize their problem-solving process.

Student Pages

Mathematical Problem Solving Challenge　　　　　　　　　　Name: ______________________

Today's exercise is about thinking creatively to solve mathematical problems. Using logic and reason to find innovative approaches to complex problems is a key skill within the mathematical sciences. To receive points, you must explain, in your own words, why your answer is correct. **It is insufficient to only provide the answer you must additionally prove that the answer you gave is correct.** Most of these problems will require more space to explain your reasoning, attach any extra work on a separate sheet of paper clearly labeled. You may work together with a partner (in fact I encourage you to do so, collaboration is one of the most productive ways to solve problems). Your competition score will come from your points earned on the questions. Remember that a complete solution is not necessary to earn points on a question and your problem solving process is the core of your work. The pair with the highest score will earn some extra credit points (determined by how well they do) and each group thereafter may earn a decreasing amount of extra credit points. You may not use any outside resources beyond the tools given to you to solve today's problems.

1. Shrinking darts is a special kind of game of darts where every time you hit the target your dart board shrinks. You start with a dart board with a radius of one, inscribed inside a square with a side length of two. Each time you throw a dart you earn a point and if you hit the circle the dart board shrinks. The game ends when you miss the circle and your score is the number of times you threw a dart (i.e. you get 1 point for playing). The circle shrinks by the following pattern: call the distance from where your dart hit from the midpoint of your circle h, draw a chord c, perpendicular to h. The length of c is the diameter of the dart board during the next round. Suppose you are a perfectly random darts player (meaning you are equally likely to hit any point on the board).



   a. On your first round how likely are you to hit the circle and continue playing? (2 points)

   b. If your first shot is very close to the edge of your circle how does that affect your next round? What about if your first shot is very close to the midpoint of the circle? (3 points)

   c. If you play this game an infinite number of times what would your average score be? (10 points)

2. There are precisely five triangles with integer sides whose perimeter is equal to their area. Each of these that you can prove you've found is worth 1 point.

3. Four bankers (Adam, Baily, Chuck, & Diane) are in charge of the accounts for four business (Eggs-R-Us, Flipper's Tuna, Golfer's Goldmine, & Hampshire Hams) with four different interest rates (5%, 6%, 7%, & 8%). Given the five statements below determine which banker works with which business at what rate. (4 points)

Banker	Business	Interest Rate
Adam		
Baily		
Chuck		
Diane		

a. Neither Golfer's Goldmine, nor Hampshire Hams earn interest at 5%.

b. Diane works with Hampshire Hams.

c. Baily either works with Eggs-R-Us or the account with a 5% interest rate.

d. Adam works with the account that is 2% less than Hampshire Ham's interest rate.

e. Baily works with the account with a 6% interest rate.

4. Prove that for the statement: $z = n^4 + a$ there are infinitely many natural numbers (a) where z is not a prime number for any natural number (n). (4 points)

5. The earth science class is going to the planetarium and there is a grant for 100$ to pay for the trip. The planetarium can hold up to 100 people. The costs of admission are 10$ per teacher, 2.50$ per chaperone, and 0.50$ per student. What combination of teachers, students, and chaperones would fill every seat and spend the whole grant? (3 points)

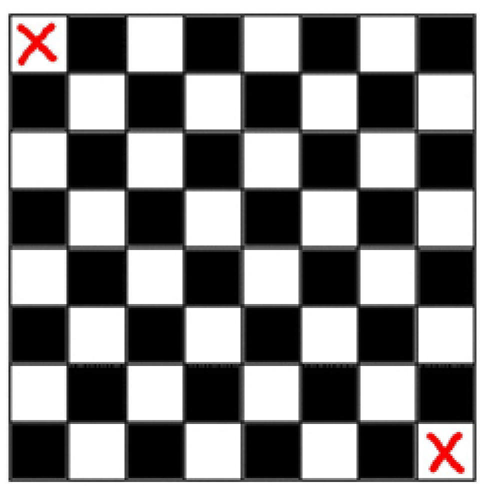
6. If a copper bowl weighs 11.5 oz. plus half its own weight, how much does it weigh? (2 points)

7. At the 2018 national astrophysicist convention many scientists met each other and shook hands. Prove that (no matter how many new handshakes occurred) the number of people who shook hands an odd number of times is even. This problem can be expressed graphically by connecting an arbitrary number of dots with an arbitrary number of (non-identical lines) and proving that for any number of dots and lines that the number of dots with an odd number of connections will always be even. (8 points)

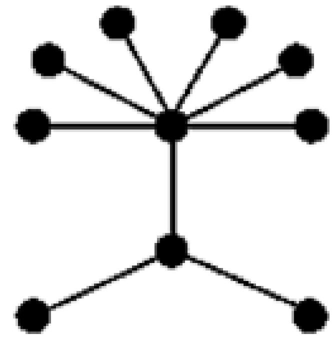
8. You have 30 green socks and 30 purple socks all mixed up in a drawer. The socks are all identical besides their color. The room is in pitch darkness and you want two matching socks. What is the smallest number of socks you must take out of the drawer in order to be certain that you have a pair that match? How do you know? (4 points)

9. Consider the integers 1 through 15 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}. Arrange this set of numbers such that each number when added with their neighboring number(s) is a perfect square (ex: 10 could go next to 6 because 16 is a perfect square). (3 points)

10. Consider a chessboard (8x8) with two diagonal corner spaces removed. Given 31 dominoes (which must always cover exactly two adjacent squares on the board) is it possible to cover all the remaining squares? If so, show how, if not prove it. (2 points)



11. A 'tree' is a type of mathematical graph composed of dots (called nodes) and connecting lines (called edges). To say a graph is homeomorphic means that (regardless of the shape the graph is drawn in) it has an equivalent configuration of nodes and edges. To say that a tree is irreducible means that the graph cannot have any loops or nodes with precisely two edges (one edge or more than two is fine). There are precisely 10 homeomorphic irreducible trees that have 10 nodes. Finding 10 is worth 2 points, showing why there are 10 (not more or less) is worth another 2 points.



XII. The following string of numbers encodes a short phrase, decode the phrase. (3 points)

2 24 13 6 1 9 4 1 6 17 6 20 17 4 17 2 7 14 24 21 15

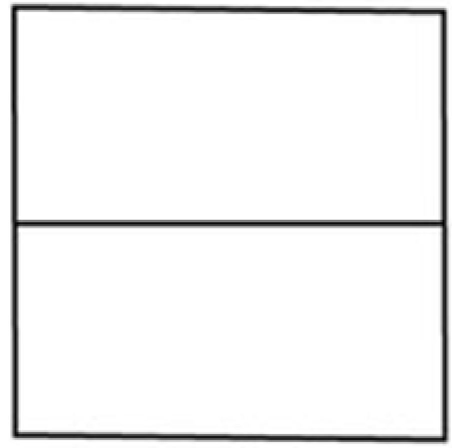
13. A group of adventurers is on a quest to sneak into a mysterious hideout. They observe a pair goblins arrive at the entrance, each time a slot opens and a guard says a number, the goblin replies with another number and is allowed to enter.

The first goblin approaches the door, the guard says "six," through the door, the goblin replies "three," and is allowed to enter. The second goblin approaches the door, the guard says "twelve," the goblin replies "six," and is ushered inside.

The adventuring parties' barbarian is certain he has found the pattern. He rushes up to the door where the guard says "twenty," through the slot. The barbarian promptly replies "ten!" and, to his chagrin, the door stays shut fast. What should he have said to be allowed inside? How do you know? (2 points)

14. Are there more, less, or the same amount of natural numbers {1,2, 3, …} or real numbers between 0 and 1? (3 points)

15. For what natural numbers of rectangles, with sides in a 2:1 ratio, is it possible to construct a square? The square may be any size and the 2:1 rectangles can be any size and any number of sizes, so long as their sides are in a 2:1 ratio. You may not overlap the rectangles. (5 points)

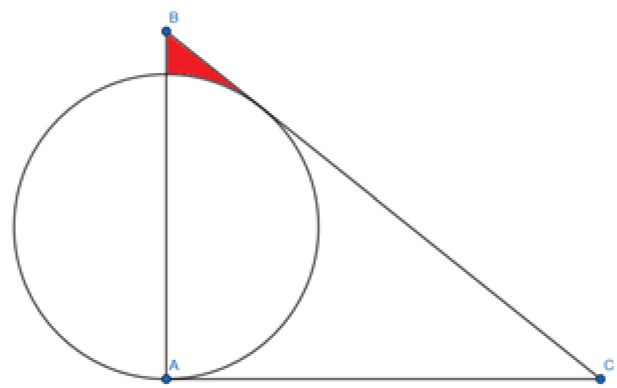


16. Josephine is playing a game with her friends where they all sit in a circle and one person is chosen to be 'it.' The person who is 'it' picks a number and starts counting from the person of their choice in a circle until they reach their number. The person they are pointing to when they reach their number is out. The person who is 'it' then continues counting from where they left off, continuing to eliminate the person who is the chosen number along the circle until there is only one person left. This person wins and is 'it' next round. Josephine's classmate Paul has won 10 games in a row, no matter how many people play it seems like Paul always knows who to start counting with and what number to count up to so that he'll win. Help Josephine turn the tables by coming up with a general pattern for the winning position given any number of players and what number should be counted by. (5 points)

17. Find the area of the shaded region given the following:

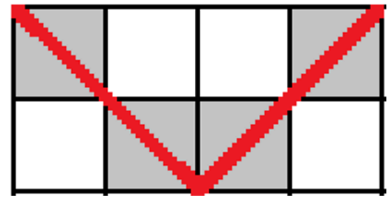
- $\angle BAC\ is\ 90°$
- The circle's midpoint lies along line AB
- The circle is tangent to line BC
- Line AB has a length of 4
- Line AC has a length of 5

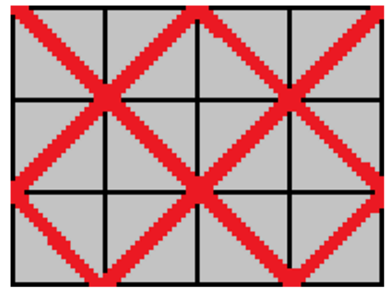
(4 points)



18. Bill was a teller working at a bank who always had trouble staying focused. Last Friday Mrs. Aubergine cashed a check for X dollars and Y cents (X and Y both being integers between 0 and 99) however Bill was careless and swapped the dollars and cents when cashing the check (giving Mrs. Aubergine Y dollars and X cents). Mrs. Aubergine left the bank without noticing the error, paid a nickel to clear her parking meter, and found that she had left exactly twice the value of her original check. What was the value of the original check? (3 points)

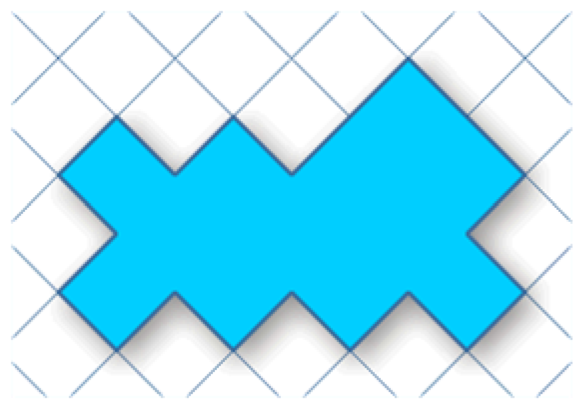
19. Consider a rectangle with sides A and B. Where A and B have natural number lengths and the rectangle is populated with a 1x1 unit grid. Suppose a laser pointer was fired from one of the rectangles corners and that anytime the beam hits a wall it bounces with an angle of repose of 90 , the laser continues bouncing until it lands in another corner. For some combinations of natural numbers, A and B this beam will only hit some of the grid squares before terminating in another corner, other combinations of A and B will always pass the beam through every grid square. What must be true about A and B for the rectangle to fall into either case? (4 points)





20. Once upon a time there was a bank robbery. Given that: Adam, Bart, and Cody are the only possible perpetrators, the robber(s) left in a truck, Cody only does crime if Adam is involved, and Bart doesn't know how to drive. Is Adam guilty? (2 points)

21. Using only one line how could you divide this figure into two identical parts? (your line does not need to be straight) (2 points)



22. Given that the distance from the midpoint of a circle to B is 6 and from B to the edge is 4 and the quadrilateral shown is a rectangle. How far is it from A to B? (2 point)